

Deductive rationality in human reasoning: Speed, validity and the assumption of truth in conditional reasoning

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Abstract

We proffer the thesis that, in the process of defeating an inference on the basis of a factual truth that falsifies it, people move from a hypothetical truth-value to a factual truth-value of the conclusion. We will present evidence that shows (a) that some people spontaneously make a truth assumption and constrain their inferences to logically valid inferences, (b) that people tend to abandon the truth-assumption when they have factual evidence to the contrary, (c) that people, however, can and do in fact reason logically when they are informed about the rules of the language game (i.e., the truth-assumption) and (d) that adhering to the truth-assumption in the face of conflicting evidence to the contrary requires an investment of time and effort. The findings are discussed in relation to contemporary theories of human reasoning.

General Introduction

We all reason: We draw inferences from the multiple sources of information we are confronted with and make decisions based on them. This allows us to move around in a changing world where the capability to comprehend the contingent nature of our environment determines for a large part our successes as an individual, as well as a species. The study of human reasoning is therefore important to advance our understanding of the general mechanisms of thought.

The turn of the century has provided the stage of a paradigm shift in human reasoning research. The nineties provided the scene for polemical debates as regards basic human reasoning competence. This basic reasoning competence (i.e., the basic machinery that allows us to draw inferences) was mostly studied by means of abstract knowledge-lean inference problem. By using arbitrary relations (e.g., ‘if the letter is an A, then the number is a 2’) no content-specific background knowledge would be triggered to influence the reasoning process towards accepting or rejecting the conclusion. Abstraction was made of the specific content of that about which people were reasoning. It is within this research milieu that theories became specified as regards human deduction. In the study of human deduction one studies necessary inferences derived from certain premises. One asks people to draw logically valid inferences, and these are defined as inferences that must be necessarily true if the premises are true. Presently there is an increasingly prominent body of evidence that shows the pervasive influence of content and belief (Cummins et al., 1991). Our beliefs are uncertain (i.e., they are true to a certain degree: e.g., even Newton’s mechanics are not universally applicable). This observation induced a shift towards the study of the subjective probabilistic properties of that about which we are reasoning as well as commonsense reasoning or reasoning under

uncertainty.

The present research is situated within this timely clash between experimental paradigms and associated theoretical approaches. Theorists sometimes like to boost the polemics between dichotomized opposites (it does make for simpler, and hence more easily publishable reading). For instance, it is claimed that theories that have focused on reasoning under certainty (i.e., deductive reasoning) are incapable of being extended to reasoning under uncertainty (i.e., probabilistic reasoning). The ‘core argument’ (Oaksford & Chater, 1998) is that common-sense reasoning is non-monotonic, whereas logic systems are monotonic: valid inferences cannot be invalidated; they remain valid. The validity of everyday inferences however would be revisable. For instance, when being given the argument:

‘If it is a bird, then it flies;

Tweety is a bird who, thus, can fly”

almost everybody will accept it. At the same time, when subsequently being told that Tweety is an ostrich, almost everybody will reject the original inference and will state that Tweety cannot fly.

The rationality debate in the cognitive science of human reasoning is partly muddled by a failure to distinguish the defeasibility of a conclusion from the non-monotonicity of an inference. For instance, Oaksford and Chater’s (1998) core argument is subverted when taking count of the distinction between truth and validity. Monotonicity concerns the validity of inferences; defeasibility concerns the truth of conclusions and this “distinction between *validity* and *truth* ... is basic to deductive logic [and] many people find the distinction difficult to grasp” (Glass & Holyoak, 1986, p. 338). The abovementioned definition of logical validity use the notion of truth but the truth of a valid conclusion is always hypothetical (*if* the premises are true, then the conclusion must also be true). The truth-value of a defeated inference however is not hypothetical. It is factual: it hinges on a factual truth (i.e., our belief, at a particular moment in time and space that something is true in the ‘real’ world).

The present study intends to show the importance of the truth-assumption and by consequence the hypothetical nature of the truth of logically valid inferences. We proffer the thesis that in the process of defeating an inference people move from a hypothetical to a factual truth-value of this conclusion. I present evidence showing (a) that at least some people make the truth-assumption and spontaneously constrain their inferences to logically valid inferences, (b) that people abandon a truth-assumption when they have factual evidence to the contrary, (c) that people, however, can and do in fact reason logically when they are informed about the rules of the language game (i.e., the truth-assumption) and (d) that adhering to the truth-assumption in

the face of conflicting evidence to the contrary requires an investment of time and effort. In the general discussion we will then return to the theoretical and conceptual issues that are touched by the evidence for people's propensity to exhibit deductive rationality in reasoning hypothetically on

is not satisfied. When the conditional enunciates a causal statement, such [p and not-q]-cases reflect *disabling conditions*. For instance, when we ask people to generate alternative causes for conditionals (1) and (3), they generally have little difficulty coming up with a relatively

Table 1
Formal representation and standard nomenclature of the four basic conditional inference problems and their default conclusions.

	Logically valid		Logically Invalid	
	Affirmation	Denial	Affirmation	Denial
<i>Premises</i>	Modus Ponens:MP	Modus Tollens:MT	Affirm consequent: AC	Denial Antecedent: DA
Major	[If p then q]	[if p then q]	[if p then q]	[if p then q]
Minor	[p]	[not-q]	[q]	[not-p]
Conclusion	[q]	[not-p]	[p]	[not-q]

the basis of a truth-assumption.

Experiment

To investigate the truth-assumption in representing the information with which we are confronted and about which we reason, and its import apropos validity and deductive rationality in human reasoning we will make use of well-known content effects in conditional reasoning. In the following I first introduce these effects. Next, I present them within a dual-processing framework. This yields some additional predictions concerning the functional and temporal relations of two conceptually distinct types of reasoning (and the corresponding distinction between hypothetical versus factual truth).

Content Effects. Table 1 presents the most commonly studied conditional inference problems. These problems are formed by an affirmation or denial of the antecedent [p] or consequent [q] of a conditional of the form [if p then q]. The content of the conditional utterance can be almost anything, e.g.:

- (1) If you turn the key, then the car will start.
 - (2) If you heat water to 100°C, then it will boil.
 - (3) If you push the brake, then the car will stop.
 - (4) If you jump into the swimming pool, then you'll get wet.
- The content effects that are observed with such realistic conditional-inference problems show that the reasoning process is strongly affected by the factual truth of the premises and/or conclusion (Politzer & Bourmaud, 2002).

At a general level the content effects are summarized as an effect of the number of factual counter-examples. For instance, the conclusions for AC and DA are falsified by situations that reflect the possibility that the antecedent is false [not-p] while the consequent is nonetheless observed [p]. When the conditional captures a causal statement, such [not-p and q]-cases reflect *alternative causes*. For instance, when we ask people to generate alternative causes for conditionals (1) and (2), they generally come up with relatively few as compared to the number of alternative causes they can generate for conditionals (3) and (4). The conclusions of MP and MT are countered by situations that represent the contingency where [p] is satisfied whereas [q]

high number of factors that might prevent the effect from occurring. For conditionals (2) and (4) people can only come up with few disabling conditions. The most robust finding in reasoning with conditionals like (1), (2), (3) and (4) above, is that people are less likely to accept MP/MT when there are many (vs. few) disablers and are less likely to accept AC/DA when there are many (vs. few) alternatives.

We proffer the thesis that belief effects in conditional reasoning and the presumed problematical nature of these effects for systems of deduction are due to a failure to play the language game of deduction. When one does not ask people to assume that the premises are true, people are not asked to reason deductively. Studies that investigate content-effects in conditional reasoning often do not even mention the truth-assumption. This implies that no implications can be drawn as regards people's deductive rationality (i.e., their propensity or capability to infer logically valid inferences). To demonstrate the importance of the truth-assumption in deduction reasoning, we decided to stress the truth-assumption and its implication that any inference made under this assumption is hypothetically true. The experiment was set up so we could compare performance on problems that did not stress the truth-assumption with problems that did stress the truth-assumption.

Expectations are relatively straightforward. When people are reasoning on the basis of the truth-assumption they will exhibit more deductive rationality as compared to situations where they reason in an unconstraint context. Deductive rationality in the present study is measured by the proportion of inferences that are valid relative to the norm of classic logic. That is, when people reason in a stressed truth-assumption context, they will endorse more logically valid MP and MT inferences. The logically invalid AC/DA arguments would not be affected by an increased impetus of the hypothetical nature of inferences made under the truth assumption. Indeed, the counterexamples to MP/MT would be excluded or impossible, if the conditional were true. However, the counter-examples to AC/DA (i.e., alternative causes) are consistent with a conditional utterance of the form [if p then q]. Indeed, the utterance "If you jump into

the swimming pool, then you'll get wet" does not say 'if and only if you jump into the swimming pool, then you'll get wet'. In sum, there should be an interaction between the logical-validity of the inference and the impetus that is placed on the truth-assumption.

Dual Processing. We noted that the present research is situated within the timely clash between experimental paradigms and associated theoretical approaches. Being faced with the task of reconciling the 'old' (deductive certainty) and the new (probabilistic uncertainty), there is an increasing popularity of so-called dual processing frameworks. There presently seems to be a growing consensus that a distinction can be made between two types of rationality, or systems of reasoning (see, e.g., Evans & Over, 1996; Johnson-Laird, 1983; Stanovich & West, 2000). Dual-process theories of reasoning draw on the distinction between, on the one hand, highly contextualized associative, heuristic, tacit, intuitive or implicit processes that are holistic, automatic, experiential in nature, and relatively undemanding of cognitive capacity and, on the other hand, de-contextualised, rule-based, analytic, explicit processes that are relatively slow, and demanding of cognitive capacity.

There is a commonality in almost all dual-processing theories. About the functional relation between the two reasoning systems it has been argued that there is a primacy of System 1 processes (Stanovich & West, 2000). Evans and Over (1996) similarly discussed the override function of System 2 (Explicit, Rationality-2 in their terminology). This functional relation parallels the distinction and relation between generate and test procedures (Chater & Oaksford, 1999) or, analogously, the conclusion formulation and validation stages proffered in the highly influential mental models approach to reasoning (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991, 2002). We can associate factual/probabilistic reasoning and hypothetical/deductive reasoning with respectively System-1 and System-2 thinking. The override function of System-2 as regards the output of System-1 consequently allows us to specify some additional expectations concerning the potential effect of stressing the truth-assumption.

In the dual-processing framework it is assumed that System-2 processes are secondary to the workings of System-1 processes. This implies that if we can inhibit system-2 thinking, the effects of its functionality will be reduced. That is, we would expect the effect of stressing the truth-assumption to be reduced under conditions that are not conducive to system-2 thinking. We can expect, the other way round, that when we can instigate system-2 thinking, the effect of its potential override function would be increased. This means that the effect of stressing the truth-assumption would be strongest under conditions that allow people to engage in the resource-dependent and time-consuming system-2 type of thinking.

We asked one group to reason as quickly as possible, thereby reducing the potential import of the system-2 thinking (see Schroyens, Schaeken, & Handley, 2003) and the expected effect of stressing the truth-assumption. A second group was asked to think carefully. Given that

people are less likely to engage in system-2 thinking under speeded inference conditions (as compared to the standard-inference conditions), we can expect that the inhibitory effect of stressing the truth-assumption will be annulled. That is, the other way around, only people who have the time and motivation to engage in system-2 type thinking will exhibit the effect of stressing the truth-assumption.

Method

Design. Participants served as their own control as regards inference type (logically valid: MP/MT vs. logically invalid: AC/DA), the number of alternative causes (few vs. many), the number of disabling conditions (few vs. many), and the impetus that was placed on the truth-assumption (no vs. strong). A between-groups factor was formed by the impetus that was placed on speed vs. accuracy.

Materials. We collected 16 conditionals utterances for which people in a pre-test were able to generate few or many alternatives and few or many disablers (see, e.g., items 1-4 presented above). The set contained four items for each of these four types of conditionals with few/many alternatives/disablers. Each conditional served as the major premise for each of the four types of inference problems (MP/MT/AC/DA, see Table 1).

The inference problems were cast into two booklets. A first booklet contained the 32 items that did not mention the truth-assumption and a second booklet with 32 other items that stressed the truth-assumption. (The specific item content was counter-balanced across the two truth-assumption conditions). Each counterbalancing set contained two items of an MP/AC/DA/MT argument about a conditional with few/many alternatives/disablers ($2 \times 4 \times 2 \times 2 = 32$). The non-stressed condition presented the problems as follows.

If you turn the key, then the car will start.

You turn the key.

It follows:

The car will start.

Participants marked their evaluation of this conclusion on a 7-point scale ranging from (1) very uncertain that the conclusion follows to (7) very certain that the conclusion follows. In the stressed truth-assumption condition the problem was presented in the following format:

If you assume that it would always be true that:

If you turn the key, then the car will start.

And you know for sure:

You turn the key.

Then it would follow:

The car will start.

Participants marked their evaluation of this conclusion on a 7-point scale ranging from (1) very uncertain that the conclusion follows if one assumes that the rule is true to (7) very certain that the conclusion follows if one assumes that the rule is true.

The instructions to the speeded inference conditions mentioned that they were to evaluate the problems fast and should not stay too long with any particular problem. After the 3rd and the 6th sheet of paper, with four problems per page, an extra page was inserted which reminded them that

they were to make their judgment ‘as quickly as possible’. In the accuracy conditions this reminder said that they were to ‘think carefully’ and that their evaluations of the conclusions should be ‘as accurate as possible’.

Procedure. Participants received both problem booklets at the beginning of the session (the standard problems first; the truth-assumption problems second). About half the students in each of two 11th and two 12th grade classes received the problems with speeded-inference instructions, whereas the other half received the accuracy instructions. The students in accuracy groups were told that the one who generated the most correct conclusions of a predetermined subset would receive 10 Euro. To the speeded groups it was said that the person who solved the problems fastest (at a minimum accuracy level) would also receive 10 Euro.

Participants. Participants were 72 11th and 12th grade student from a Belgian, Flemish high school. Thirty-four students received the speeded-inference instructions; the remaining 38 pupils ended up in the accuracy conditions.

Results

Certainty ratings (1-7) were transformed to the [0,1] probability interval and submitted to analyses of variance. Figure 1 presents the effect of alternatives on the logically invalid inferences (AC/DA), and the effect of disablers on the logically valid inferences (MP/MT) in the standard conditions that do not mention the truth assumption. These standard problems replicate the standard findings. First, the number of disablers affected the certainty ratings of the logically valid inferences: Participants are more certain that the conclusion follows when there are few counterexamples, .81 vs. .69; $F(1,70) = 53.06, p < .01$. Second, the invalid inferences also showed the standard counterexample effect of few vs. many alternatives: Participants rate the conclusions less certain when more counterexamples can be found for it, .81 vs. .60; $F(1,70) = 153.75, p < .01$. Figure 1 also shows that the counterexample effect is larger on the logically invalid inference, as compared to the logically valid inferences; $F(1,70) = 21.38, p < .01$.

Figure 2 shows the size of the counterexample effect (few vs. many) as a function of the timing constraint, logical validity and the assumption of truth. Figure 2 clearly shows that the counterexample effect on the logically valid inferences is reduced when people make the truth-assumption, $F(1,70) = 14.21, p < .001$, but only so when individuals reason without a timing constraint and focus on accuracy, $F(1,70) = 19.67, p < .01$. The counterexample effect does not approach significance in this condition, .866 vs. .853. The interaction between speed and truth at the level of the valid inferences was significant, $F(1,70) = 5.41, p < .05$. No such interaction was observed at

Figure 1

Certainty ratings of the logically valid and invalid arguments under standard conditions that do not mention the truth assumption.

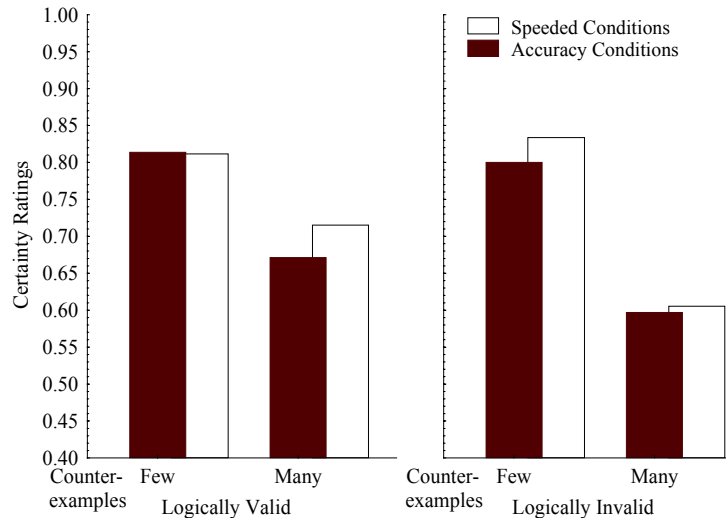
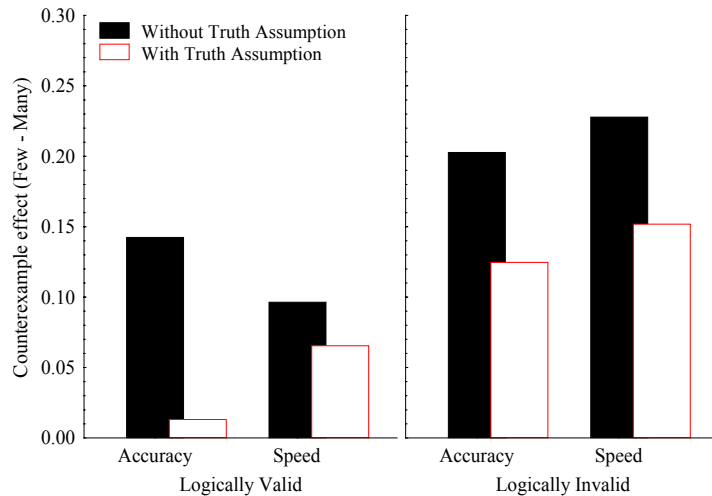


Figure 2

Counterexample effects on the logically valid and invalid arguments as a function of a timing constraint and the explicit presence of the truth assumption.



the level of the invalid inferences ($F = .003$), and the third-level interaction indeed tended to approach statistical significance, $F(1,70) = 2.85, p < .10$. Specific comparisons showed that, as expected, the counterexample effect on the valid inferences re-appears when people evaluate the conclusions as fast as possible, .873 vs. .808; $F(1,70) = 9.66, p < .01$. That is, stressing the truth-assumption does not reduce the counterexample effect on the logically valid inferences when people are reasoning under a timing constraint ($F < 1$). At the level of the logically invalid inferences, we see an overall reduction of the counterexample effects, $F(1,70) = 13.22, p < .01$. This might suggest that stressing the truth-assumption tends to

induce an overall inhibition of background knowledge. The fact that the truth-assumption effect on the valid inferences depends on the timing constraint tells us that this is not the entire story. Also, as noted before, the counterexample effects on the valid inferences are completely annulled when the truth-assumption is stressed (under accuracy conditions), whereas Figure 2 shows that they are still very much present on the invalid inferences under the same conditions. The reduced counterexample effects on the invalid inferences presented under truth conditions concurs with the idea that some people adopt a bi-conditional interpretation of 'if'. The alternative causes are then theoretically or hypothetically (i.e., under the assumption of the truth of the utterance) impossible.

Discussion

Our findings corroborate several of the claims we have made regarding deductive rationality in human reasoning. First, in order to reason deductively one has to make and adhere to the truth-assumption. We observed that the counter-example effect on the logically valid inferences is indeed smaller than that on the logically invalid inferences. The counter-examples to logically valid inferences are indeed (hypothetically) impossible – this is actually why these inferences are logically valid. Second, though we have evidence that some people spontaneously exhibit deductive rationality in adhering to the truth-assumption, other people clearly abandon the truth-assumption in the light of factual evidence to the contrary. The probabilistic counterexample effects on the logically valid inferences attest to this.

The speed/accuracy manipulation and the effects of stressing the truth-assumption provide strong support to our analyses of deductive rationality within a dual processing scheme. First, the overall increase in deductive rationality (as measured by the increase in the certainty ratings of the logically valid inferences) under conditions that stress the truth-assumption lends support the centrality of the truth-assumption in the notion of logical validity and human deductive reasoning. Second, the annulment of the counter-example effects on the logically valid inferences under conditions that make it clear that the truth of inferences about factually false utterances is a hypothetical truth, is also in agreement with the thesis that people inhibit factual knowledge that conflicts with the hypothetical truth of the utterances people reason about. Third, the dependency of the counter-example effect annulment on the time and effort people take to provide an evaluation of the conditional inferences, concurs with (and was predicted on the basis of) the thesis that probabilistic content-driven reasoning is primary to the effortful abstract, analytic hypothetical reasoning processes that can serve to override the output of the fast and frugal heuristic processes.

General Discussion

Our study shows the import and importance of the truth-assumption as regards deductive rationality in human reasoning. In the current general discussion we will touch upon some wider theoretical and conceptual issues. We will first consider the rational basis for the truth-assumption. Next, we will consider the import and importance of the

truth-assumption as regards arguments that have been made in discussions of the non-monotonic and/or defeasible nature of human reasoning.

An Implicit vs. Explicit Truth-Assumption. We found support for thesis that at least some people make the truth-assumption and actually stick to it. It remains the case, however, that the majority of people will abandon the truth-assumption. The sizable counterexample effects on the logically valid inferences evidence this. One can only claim that the truth-assumption is abandoned when it is made in the first place. The question that then arises is whether those people who do not follow the truth-assumption (by taking count of factual knowledge to the contrary) actually made it in the first place.

It is our contention that when people form a representation of the utterances they are confronted with, they initially and implicitly make the assumption that the proposition expressed by it is true. This is in accordance with the Gricean maxims of conversation (which by themselves are related to Kant's four a priori categories of quantity, quality, relevance and modality): we generally assume/ensure that our or the speaker's contribution is truthful, relevant and as informative as possible, though not more detailed than required by the context (Grice, 1975). The truth-assumption is an implicit assumption (see, e.g., Schroyens, Schaeken, & d'Ydewalle, 1999). It is partly because it is an implicit assumption (at least to start with) that it is easily abandoned. The rational basis of the truth-assumption can be found in the idea of bounded rationality or cognitive economy. There is a representational cost attached to considering all possibilities, both true and false.

Most current theories presume the truth-assumption. This is not very surprising when one considers that truth is ontologically primordial to falsity: Non-truth presumes truth – as non-being presumes being. The mental-models theory (Johnson-Laird & Byrne, 2002) is the single one theory that is most explicit in invoking the truth-assumption. Indeed, it forms the basis of the truth-principle as regards the representation of the meaning of conditionals of the form [if p then q]. This principle states that people initially represent only represent true possibilities. Oaksford, Chater, & Larkin (2000) seems to have the only theory for which it is difficult to see whether it incorporates the truth-assumption. They do not seem to distinguish true from false utterances. There are only degrees of truth (i.e., probabilities). This restriction to factual truth (verisimilitude) is problematical because there is plenty of evidence that shows that people can reason hypothetically and deductively.

Truth, Validity and Non-Monotonic Reasoning. We situated the present study within the timely clash between paradigms focusing on deductive or probabilistic reasoning and presented the core argument that is made against logic theories. Theories of human deduction would not be capable to cope with the defeasible nature of human reasoning.

Our introductory analyses of the core argument against mental logic have shown that the issues are more complex: The defeasibility of a conclusion does not necessarily imply the non-monotonicity of an inference. Let us reiterate our

arguments against the claim that logic is in trouble because it is monotonic, while commonsense reasoning would not be. Indeed, we have come to the somewhat controversial conclusion that it remains an open question whether commonsense reasoning is non-monotonic (even though we know it is defeasible).

We know that the counterexamples to the Modus Ponens argument (MP: if p then q, p, therefore q) are cases that naive reasoners (as opposed to logicians) consider impossible if the conditional utterance is true (Evans, Ellis, & Newstead, 1996). When they assume that [if p then q] is true, most people generally judge that it would be impossible that there are [p and not-q]-contingencies: situations wherein the consequent does not follow from the antecedent. In short, when people defeat a logically valid inference this simply indicates that peoples' intuitive notion of validity does not match that of logical validity. The pervasive 'belief effects' show that reasoners are much more concerned with the factual truth of a conclusion (Tweety the ostrich does not fly), as compared to the hypothetical truth of such conclusions (if it were true that all birds fly then Tweety the ostrich would fly).

Since logical validity encompasses the truth assumption, defeating a necessary inference marks the abandonment of this truth-assumption. By consequence it remains undetermined whether people have reasoned non-monotonically (i.e., revised a judgment of logical validity into a judgment of logical invalidity). When we assume, for arguments sake, that people actually aim to derive logically valid inferences, the defeasibility of inferred inferences shows that people shift from one notion of validity (i.e., logical validity, which includes the truth-assumption) to another notion of validity (let us call it 'psychological validity', which gives more weight to factual truth and allows a truth-assumption to be annulled). It seems one succumbs to the fallacy of equivocating two distinct concepts (logical and psychological validity), when defeasibility of an inference is taken to indicate non-monotonicity of human reasoning.

Because classic logic is monotonic while everyday reasoning is presumably non-monotonic (or at least defeasible), it has been stated that neither mental-models theories nor mental-logic theories are capable of explaining common-sense reasoning. It is hard to see why polemics have been created when defeating inferences is actually at the heart of mental models theory. Mental-models theory holds to a three-stage processing scheme. People first generate initial (incomplete) representations of what they think is possible if the premise are true (model-construction); they then integrate the representation of the multiple source of information that form a reasoning problem (model-integration). This allows them to generate a putative conclusion, which, third and most importantly, at least some people at least sometimes attempt to test by looking for a counterexample. A conclusion is rejected and/or modified in the light of conflicting information. That is, defeasible reasoning is in no way beyond the reach of mental-models theory, quite on the contrary:

"It is worth given up, not the thesis that human beings are capable of rational thought, but the idea that what underlies

this ability is a mental logic. There can be reasoning without logic. More surprisingly, perhaps, there can be valid reasoning without logic" (Johnson-Laird, 1983, p. 40).

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