Chapter 11

Symmetry-Breaking and the Contextual Emergence of Human Multiagent Coordination and Social Activity

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Here we review a range of interpersonal and multiagent phenomena that demonstrate how the formal and conceptual principles of symmetry, and spontaneous and explicit symmetry-breaking, can be employed to investigate, understand, and model the lawful dynamics that underlie self-organized social action and behavioral coordination. In doing so, we provided a brief introduction to group theory and discuss how symmetry groups can be used to predict and explain the patterns of multiagent coordination that are possible within a given task context. Finally, we argue that the theoretical principles of symmetry and symmetry-breaking provide an ideal and highly generalizable framework for understanding the behavioral order that characterizes everyday social activity.

1. Introduction

> "*. . . it is asymmetry that creates phenomena.*"
> (English translation) Pierre Curie (1894)

> "*When certain effects show a certain asymmetry, this asymmetry must be found in the causes which gave rise to it.*"
> (English translation) Pierre Curie (1894)

Human behavior is inherently social. From navigating a crowed sidewalk, to clearing a dinner table with family members, to playing a competitive sports game like tennis or rugby, how we move and act during these social contexts is influenced by the behavior of those around us. The aim of this chapter is to propose a general theory for how we might understand

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the emergence, stability, and dynamic organization of such coordinated social activity. In particular, we aim to delineate a theoretical framework for understanding what creates and determines the behavioral patterns that are possible within a given environmental or social context, one that is indifferent to the particulars of the behavioral phenomena in question and can therefore provide a generalized account of the coordinated behavioral activity that occurs across all manner of behavioral tasks. The theoretical account being proposed is based on the conceptual and formal principles of symmetry and the theory of symmetry-breaking, and is motivated by a number of recent research findings and the famous words of Pierre Curie (1894) cited above, the second of which is commonly referred to as Curie's Principle. In short, our aim is to demonstrate how the behavioral dynamics and collective organization of multiagent coordination is defined by the symmetries, asymmetries, and symmetry-breaking events of environmentally situated social activity.

Before jumping headlong into the details of how symmetry and symmetry-breaking can provide a theoretical account of what, why, and when different patterns of multiagent behavioral coordination emerge, consider the following example that many of us can identify with. Imagine an individual at a wedding, seated at one of those circular and flawlessly set dinner tables, with the glasses positioned in perfect symmetry around the table. In this situation, the seated individual is often left with the question as to which of the two glasses within reach is theirs–the glass to the right or the glass to the left? That is, the individual, as well as the collective group of individuals seated at the table, exist at the precipice of a right- or left-glass state (much like Buridan's Ass[1] stuck between two identical bales of hay). This dilemma is easily solved, of course, as soon as any one individual at the table has the courage to grasp a glass. For instance, if any individual chooses a glass to his or her right the collective order of the group immediately collapses to a right-glass state, with every individual in the group choosing the glass to the right. This is true even for those individuals not yet seated at the table, in that the symmetry-breaking act of one individual's glass selection specifies the behavioral choice for all current

[1]Named after the French philosopher Jean Buridan, the Buridan's Ass paradox refers to the situation where an Ass (Donkey) finds itself equidistant between two bales of hay and because the Ass is continuously pulled in equal measure to both bales of hay is unable to decide which bale of hay to move towards and eventually starves to death (unless, for example, the Ass is right or left-legged, or there is more wind coming from the west or east).

and future individuals seated at the table.

This example highlights the deep and meaningful points that Pierre Curie was attempting to make in the introductory quotes. The first is that the order of behavior emerges from symmetry-breaking events. Here the term *order* simply refers to the collective organization or observed patterning of a system's behavior over time. Within the context of the "whose glass is whose" dilemma, the selection of a specific glass by any one individual operates to break the symmetric arrangement of the glasses as a whole and, thus, defines the organization or pattern of the table's group behavior[2]. The second point, also highlighted in the above example, is that the behavioral order (patterning) of coordinated social activity is determined by the (a)symmetries that define an agent-environment task context. In more general terms, this implies that the symmetry of the effect is written in the symmetry of the causes that bring about that effect (Stewart and Golubitsky, 1992/2011).

As we will explain in more detail below, by symmetries, we are referring to the task-relevant agent and environmental properties that are equivalent or invariant with respect to a given transformation. These properties could refer to the physical or informational aspects of the task environment, the biomechanical or perceptual-motor abilities of the co-acting agents, or the agents' intentional states, goals, or psychological dispositions. Accordingly, although the focus of the current chapter is on the application of symmetry principles for understanding the organizational structure that characterizes social action and multiagent coordination, we foresee that the conceptual and formal aspects of symmetry and symmetry-breaking can be employed to understand all forms of human perception, action, and cognition. Why then do we focus on social action? Well, for no other reason than the ideas discussed here have emerged out of our work examining the behavioral dynamics of social coordination and joint-action.

With this in mind, the chapter is structured as follows. In the first half of the chapter we briefly detail the formal and theoretical concepts of symmetry and symmetry-breaking. Then, using several empirical examples, we illustrate how the principles of symmetry and symmetry-breaking can be employed to understand the dynamics and behavioral order of social, multiagent coordination. Finally, we conclude the chapter by proposing that

[2]The "whose glass is whose" dilemma was first described to the authors of this chapter by Robert Shaw, who has also employed this example in his own theorizing about how the principles of symmetry underlie human perception and action (Shaw, McIntyre and Mace, 1974; Warren and Shaw, 1985).

the formal and conceptual principles of symmetry and symmetry-breaking provide a general theory for modeling and understanding the behavioral dynamics (Warren, 2006) of social coordination and multiagent activity, as well as human behavior in general.

2. Symmetry

Most of us have a good idea of what the term symmetry means, typically defining symmetry as meaning some type of correspondence in the shape or configuration of an image or object. Accordingly, people often refer to the bilateral or mirror symmetry of the human face or body when defining the term. Others often refer to numerous geometric shapes, such as a circle, star, or octagon as having symmetry, stating how these shapes look exactly the same when rotated or reflected in certain ways. Although this commonplace understanding of symmetry is fundamentally correct, the principle of symmetry is one that applies to much more than the shapes or patterns of objects and images. Indeed, the principle of symmetry is a much more abstract concept that can be applied to understanding the regularity or order of anything, from mathematical functions, to chemical compounds and crystals, to quantum mechanics and the fundamental conservation laws of physics.

To understand this more general notion of symmetry, we can define the term symmetry as simply referring to an equivalence or invariance of some kind, given some form of transformation. The vagueness of this definition is deliberate, in that 'some kind' of equivalence or invariance and 'some form' of transformation could stand for almost anything. For example, an equivalence or invariance could refer to the fact that a square looks exactly the same when rotated by 90 degrees or reflected about a midline axis, or that a stack of three tennis balls in a cylindrical tennis ball container would look the same if one permutes the order in which the tennis balls are stacked on top of each other (the original and permuted tennis ball arrangements would be indistinguishable from each other), or to use a more large scale example (one fundamental to scientific inquiry), finding that the results of an experiment conducted on Wednesday September 8$^{\text{th}}$ 2014 at 9 am at the Psychology Department at the University of Cincinnati in Ohio, USA, replicate the results obtained using the same experimental method on Friday March 12th 2013 at 1 pm at the Psychology Department at University of Auckland in New Zealand, which also replicate the results obtained from the Department of Cognitive and Information Sciences at the University of California Merced, on Monday November 21st, 2012 at 5:30 pm.

As is the case in each of the previous examples, the central aspect of what makes something symmetric, is whether the specific phenomena or observable in question is indistinguishable with respect to a specific type or set of transformations. With this more abstract definition of symmetry in hand, one can also start to see how the symmetry of an object, phenomena, or thing can be quantified, in that some object, phenomena, or thing may have more or less symmetry than some other object, phenomena, or thing. To use a common example, consider the geometric shapes of a circle and an equilateral triangle. There are an infinite number of rotations and midpoint reflections that leave a circle looking the same, whereas there are only 3 rotations and 3 reflections that leave an equilateral triangle the same (see Figure 1). Therefore a circle has a larger, continuous set, or group, of symmetries than an equilateral triangle, which only has a discrete set symmetry transformations. A square also has a discrete set of symmetries, 8 to be exact: four rotations, two mirror reflections, and two diagonal reflections. Therefore, a square also has a greater number of symmetries than an equilateral triangle, but still much less than the symmetries of a circle.

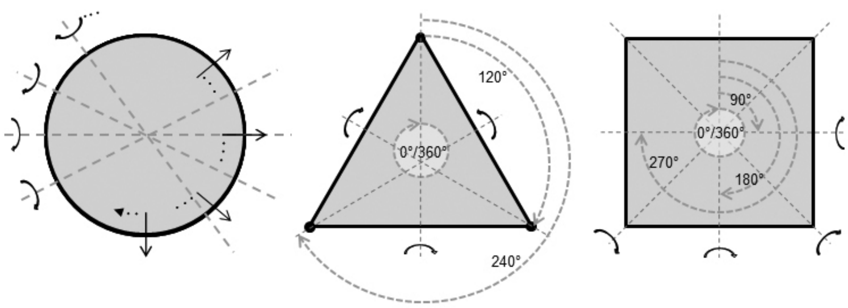


Fig. 1. Symmetries of a circle, equilateral triangle, and square. (*left*) A circle has an infinite (continuous) number of rotations and reflections about center point axes that are captured by the orthogonal group O(2). (*middle*) An equilateral triangle has 6 discrete symmetries, three rotations = $(0°$ or $360°, 120°, 240°)$ and three reflections, that are captured by the dihedral group, D_3. (*right*) A square has 8 symmetries, 4 rotations = $(0°$ or $360°, 90°, 180°, 270°)$ and 4 reflections, that are captured by the dihedral group, D_4.

2.1. *Group theory*

A conceptual understanding of symmetry is by itself a powerful tool for understanding the order and regularity of nature, as well as the significant role that symmetry plays in scientific investigation and explanation (Rosen, 1995, 1975/2012), and is an approach that we will employ in explicating the role of symmetry and symmetry-breaking in the dynamics of social coordination throughout this chapter. In many instances, however, adopting a formal, quantitative approach to symmetry is necessary to uncover what and why particular patterns or orders emerge and whether such patterns of order characterize general principles of both ontological and phenomenological organization. The formal approach to symmetry is achieved by the use of special types of sets, called groups, and the related theory of such sets called Group Theory. In essence, groups provide a way of quantifying the symmetry of something, whereby the symmetry group of an object, phenomena, or thing corresponds to a closed set of transformations that leave the system unchanged with respect to some defined property. More specifically, a symmetry group is a set, G, of symmetry transformations that together with a group operator (a binary rule of element combination) satisfy the following four group properties:

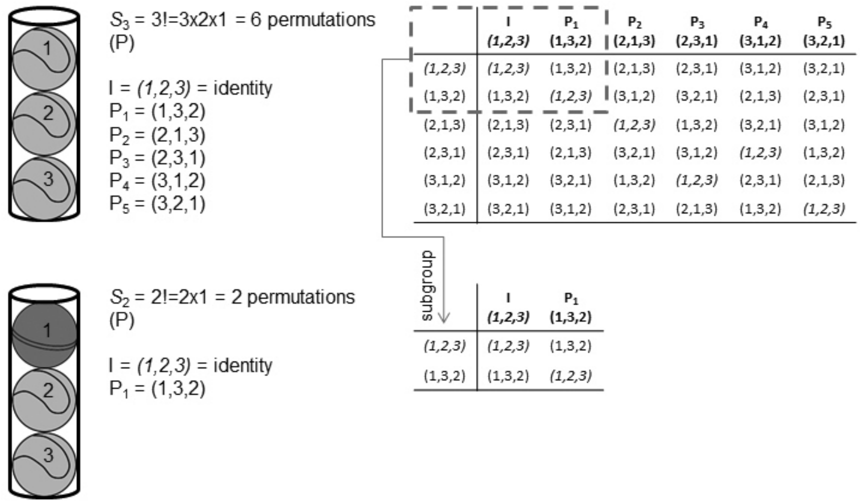
(1) The group, G, includes an *identity* transformation (often denoted as I and e), such that for every element, g$\in$ G, I$\cdot$g = g$\cdot$I = g. In many instances, the identity transformation essentially corresponds to the 'do nothing' transformation. For instance, a rotation by 0° (or equivalently by 360°) corresponds to the identity transformation for an equilateral triangle or a square.
(2) The transformation elements in G are *associative*. That is, if g1, g2 and g3 are elements of G, then (g1$\cdot$g2)$\cdot$g3 = g1$\cdot$(g2$\cdot$g3). That is, regrouping the elements with respect to the group operator (but leaving the order the same) doesn't change the outcome. Note, however, that groups do not have to be commutative.
(3) For each transformation element in G there exists a unique *inverse*, such that for every element, g, in G, there exists a g^{-1}, whereby $\mathrm{g}\cdot\mathrm{g}^{-1} = \mathrm{g}^{-1}\cdot\mathrm{g} = I$. For an equilateral triangle, for example, the inverse of the 120° rotation is a rotation by 240° (which is equivalent to −120°), such that performing both transformations is the same as rotating the triangle by 0° (i.e., $120° + 240° = 360° = 0°$).
(4) For any two transformation elements, g1 and g2, in G, the product

operation of g1 and g2 equals another element of G; the *group is closed*. To use the equilateral triangle as the paradigmatic example again, this means that for any combination of rotations and reflections one performs, the result must be equal to one of the six potential transformations in the group as a whole. For instance, the outcome of rotating an equilateral triangle by 240° and then flipping it about the vertical midline axis is equal to simply rotating the triangle by 120°.

There are many different types of symmetry groups that satisfy these four properties, from finite, single element groups that contain only an identity element, I, (i.e., trivial groups) to groups that contain an infinite number of elements. As noted above, the symmetry group of a circle is an infinite or continuous group and corresponds to the orthogonal group O(2). The symmetry groups for an equilateral triangle and a square are examples of finite groups and correspond to the dihedral groups D_3 and D_4, respectively (note that a dihedral group, D_n, is the symmetry group of an n sided polygon, such that the number of elements in, D_n, is $2n$). The above example of 3 tennis balls stacked in a cylindrical container is also an example of a finite group and can be captured by the permutation group S_3, which means that there is n-factorial ($3! = 3 \times 2 \times 1 = 6$) ways of indistinguishably rearranging the tennis balls in the container (see Figure 2).

The simplest way to verify whether a finite set of symmetry transformations forms a group is to compose the transformations into a multiplication table (sometimes referred to as a group table)[3]. To illustrate this, we have written out the multiplication table for the permutation group S_3 in Figure 2, which as just stated above can be used to capture the symmetry group of 3 tennis balls stacked within a cylindrical container (or any other system that has three elements that are symmetric with respect to permutation). From this table, one can easily determine that the six possible permutations of S_3 form a group because: (i) there is an identity element; (ii) the composition of the elements is associative; (iii) every element has an unique inverse specified by the fact that there is exactly one identity element in each row and column; and (iv) there are no empty cells in the table and thus the set is closed.

[3]For infinite or continuous groups one needs to formulate a general rule to express the closed composition of group elements (Rosen, 1975/2012).



	I (1,2,3)	P_1 (1,3,2)	P_2 (2,1,3)	P_3 (2,3,1)	P_4 (3,1,2)	P_5 (3,2,1)
(1,2,3)	(1,2,3)	(1,3,2)	(2,1,3)	(2,3,1)	(3,1,2)	(3,2,1)
(1,3,2)	(1,3,2)	(1,2,3)	(3,1,2)	(3,2,1)	(2,1,3)	(2,3,1)
(2,1,3)	(2,1,3)	(2,3,1)	(1,2,3)	(1,3,2)	(3,2,1)	(3,1,2)
(2,3,1)	(2,3,1)	(2,1,3)	(3,2,1)	(3,1,2)	(1,2,3)	(1,3,2)
(3,1,2)	(3,1,2)	(3,2,1)	(1,3,2)	(1,2,3)	(2,3,1)	(2,1,3)
(3,2,1)	(3,2,1)	(3,1,2)	(2,3,1)	(2,1,3)	(1,3,2)	(1,2,3)

	I (1,2,3)	P_1 (1,3,2)
(1,2,3)	(1,2,3)	(1,3,2)
(1,3,2)	(1,3,2)	(1,2,3)

Fig. 2. An illustration of how the symmetry group S_3 can capture the spatial transformations (permutations) of three identical objects, in this case three tennis balls stacked together in a cylindrical container. (*top panel*) The six different permutations captured by S_3 and the corresponding multiplication table. As per convention, the composition is column first then row, such that multiplication involves the row element on the left and the column element on the right of the group operator. (*bottom panel*) An example of how replacing one of the tennis balls with a different, non-identical ball (i.e., a cricket ball) reduces the symmetry group of the three objects to the permutation group S_2, which is a proper subgroup of S_3.

Figure 2 also illustrates the notion of a subgroup. A subgroup is a subset of elements from a group, that is itself a group (meets the four group properties), and includes the group's identity element. The full group and the group's identity element are always trivial subgroups of a group, with the former also termed an improper subgroup. All other subgroups of a group are non-trivial, proper subgroups. Finally, the number of elements or order of any subgroup must be a divisor of the number of elements that make up the higher order group. With respect to the permutation group S_3, there are two other S_2 subgroups in addition to the one illustrated in Figure 2, (1,2,3), (1,3,2), namely (1,2,3), (3,2,1) and (1,2,3), (2,1,3). The alternating (cyclic) subgroup (1,2,3), (2,3,1), (3,1,2) denoted by A_3 is also a subgroup of S_3.

A more comprehensive review of Group Theory and symmetry groups is beyond the scope of this chapter (see Rosen, 1995, 1975/2012; Weyl, 1952, for excellent overviews of group theory). We introduce the basic tenets of Group Theory here, however, as we will, to a limited extent, employ the symmetry group formalism to elucidate the role of symmetry in some of the examples employed later in this chapter. We also feel that it is important to emphasize the significance of Group Theory for understanding and defining the symmetry of something, including how the symmetries of one thing are related to the symmetries of something else. Indeed, Group Theory cannot only be used to formally define the symmetries of an object, phenomena, or thing, but can also be employed to determine whether the symmetries of two or more different objects, phenomena, or things are equivalent (see Figure 3). Accordingly, one can use symmetry groups to generalize a theoretical understanding of the ordered relations that exist or emerge across completely different objects, phenomena, or things, so long as those objects, phenomena, or things are defined by the same symmetry group or by an isomorphic or homomorphic symmetry group (i.e., when the symmetry groups can be shown to have corresponding structures).

3. Symmetry-Breaking

An aspect of symmetry that is often overlooked is that symmetry implies asymmetry. More specifically, a symmetry or symmetry transformation can only be defined with respect to something that is not symmetric (i.e., a gauge that captures when a transformation has occurred). That is, equality can only be defined with respect to inequality; invariance can only be defined in relation to variance. Accordingly, symmetry and asymmetry are duals (Kugler and Shaw, 1990; Shaw, Kugler and Kinsella-Shaw, 1990), (Shaw *et al.*, 1974; Warren and Shaw, 1985). Note that complete symmetry is therefore equivalent to complete asymmetry, both of which correspond to a complete lack of something, to a nothing or an absolute absence of differentiation and discrimination. In order to fully comprehend this later point, imagine if all the matter and energy in the universe was maximally and evenly distributed throughout the universe (the universe was in a state of maximum entropy). In this state of perfect symmetry the universe would look equivalent to an observer no matter what point in space (or time) the observer was located, leaving the observer simultaneously anywhere, everywhere, and nowhere.

To use a more tangible example, consider a perfect sphere of uniform color floating in space. The symmetries of such a sphere are the continuous (infinite) set of rotations and reflections about axes that pass through the sphere's center and are captured by the orthogonal group O(3). The rota-

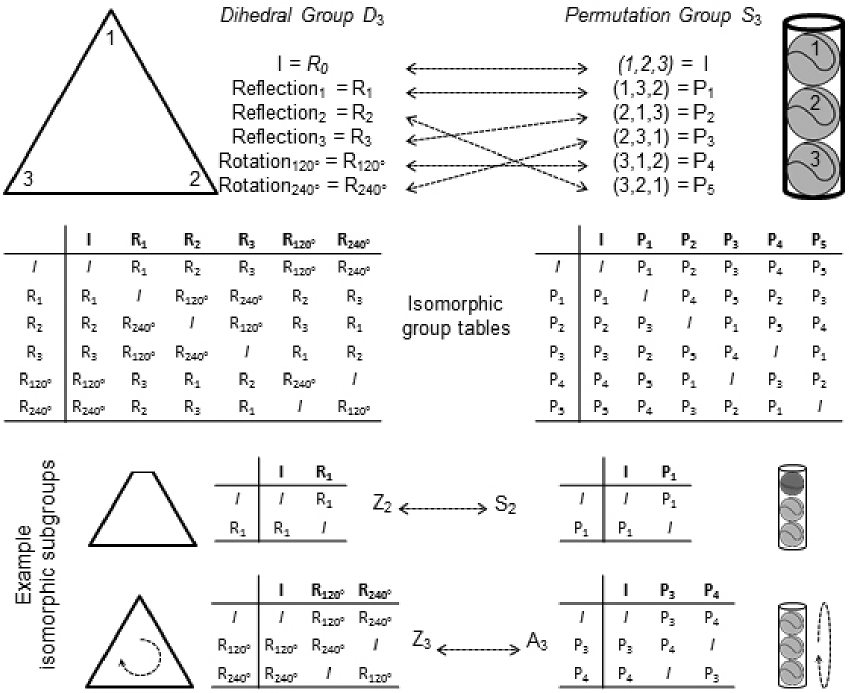


Fig. 3. An example of group isomorphism. (*top panel*) The isomorphic relationship between the Dihedral group, D_3, and the permutation group, S_3. The arrows reveal the one-to-one correspondence between the six elements in each group. (*middle panel*) The isomorphic group tables. (*bottom panel*) Illustration of the isomorphic relationships between two proper subgroups of D_3 and S_3. The first between the 2-order cyclic subgroups Z_2 and S_2, and the second between the 3-order cyclic subgroups Z_3 and A_3.

tional symmetry of this uniform sphere floating in space, however, means that one is unable to tell if the sphere is stationary or spinning around a center axis point. Anyone who has ever created a uniform grey sphere within an empty 3-dimensional graphics environment (i.e., openGL, or a modern graphics programming engine such as Unity 3D) has probably experienced this first hand. Unless one adds a non-uniform texture to the sphere, one is unable to perceive whether the sphere is still or spinning. As illustrated in Figure 4, however, as soon as a non-uniform texture is added to the sphere the rotational symmetry of the sphere is broken and one can immediately perceive whether the sphere is spinning or not. In fact, the addition of a symmetry-breaking texture means that one is also able to

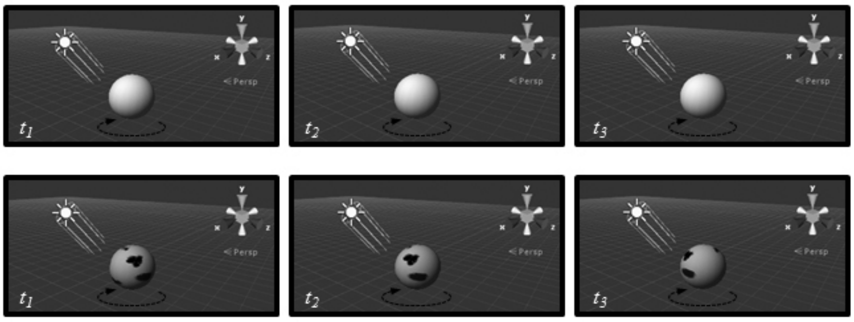


Fig. 4. Illustration of how the continuous rotational symmetry of a sphere results in any rotational movement being undetectable (essentially invisible), unless a non-uniform (symmetry-breaking) texture is added. The texture induced break in symmetry creates information about the speed and direction of spherical rotation. (*top panel*) A sequence of three fixed interval (time, t) images of a uniform rotating sphere. In each image, the sphere looks identical; the rotating sphere appears to be stationary. (*bottom panel*) A sequence of three fixed interval images of a non-uniform, textured rotating sphere. In these images, the sphere's rotational direction is apparent. The images were created using Unity3D. The dashed arrow represents the direction of rotation. The white light/sun symbol and emanating lines indicate the direction of the light source employed. The perspective shape in the top right of each image indicates the Euclidean x, y, z directions of the 3-dimensional graphics environment.

perceive what axis the sphere is rotating around, as well as the speed of rotation. The break in symmetry, therefore, creates information about the sphere's behavioral order, in this case information that specifies its speed and direction of rotation. Note again the duality of symmetry and asymmetry that this implies. The symmetry-breaking texture creates information about a local transformation (variance), the sphere's direction and speed of rotation, as a direct function of the structural invariance, or symmetry, of the spherical shape (i.e., its constancy of shape) and the global invariance of the background environment[4]. Again, symmetry and asymmetry,

[4]The deep relationship between symmetry, symmetry-breaking and information, including how symmetry and asymmetry (variance and invariance) underlie the perception of environmental objects, surfaces, and events are not discussed here. These relationships, however, are fundamental tenets of the Ecological approach to perception and the theory of direct perception proposed by James J. Gibson (1979) and subsequently development by Shaw and Turvey and others (see e.g., Michaels and Carello, 1981; Shaw, McIntyre and Mace, 1974; Shaw and Pittenger, 1978; Shaw, Turvey and Mace, 1982; Turvey and Shaw, 1999; Warren and Shaw, 1985). Recent work by Li, Sawada, Shi, Steinman and Pizlo (2013) and Pizlo *et al.* (2014) has also demonstrated the importance of symmetry for a computational approach to shape and object perception.

invariance and variance go hand-in-hand.

The duality of symmetry and asymmetry and the knowledge that complete and absolute symmetry corresponds to the absence of differentiation (the lack of something), implies that the existence of order is dependent on symmetry being broken. This is what Pierre Curie meant by the statement, *"asymmetry [broken symmetry] is what creates phenomena"* (see Brading and Castellani, 2003; Castellani, 2003, for a more detailed discussion of Pierre Curie's symmetry arguments). Indeed, any symmetry that can be perceived, observed or measured only exists as a result of a break in symmetry (and vice versa). Symmetry-breaking is therefore fundamental for the emergence, detection, and manifestation of structure, organization, and behavioral order. It is imperative that one understands, however, that the terms *broken symmetry* and *symmetry-breaking* do not refer to the absence of symmetry. Rather these terms refer to the reduction or redistribution of symmetry with respect to some higher order[5] (often "hidden") symmetry (Castellani, 2003; Rosen, 1995; Stewart and Golubitsky, 1992/2011). Indeed, symmetry-breaking is entailed by symmetry and reveals symmetry, as the sphere example highlights. With that said, we can now detail the two general ways in which symmetry breaking can occur: *spontaneously* and *explicitly*.

3.1. *Spontaneous symmetry-breaking*

Spontaneous symmetry-breaking refers to the situation where the process, parameter, or event that breaks the symmetry of a system is entailed by the symmetry of the system itself (assuming one defines the system in question appropriately). As implied by Curie's Principle, the symmetry of a symmetry-breaking effect is defined by the symmetry of the causes that bring about that effect. Employing an example from classical physics, consider a circular rod made of relatively rigid foam that is of sufficient length that one could apply a compressing force to the two ends of the rod with one's hands. The foam rod will remain straight, invariant to all rotations around its midline axis, when low compression forces are applied. However, with a critical magnitude of force, this symmetric equilibrium state will become unstable such that any magnitude of force above this critical point will cause the rod to buckle (bend). That is, the continuous rotational sym-

[5] Here the term *order* is used in reference to the number of elements in a symmetry group. That is, the number of symmetry transformations that leave an object, phenomena or thing unchanged.

metry of the rod will break, with the rod adopting a new asymmetric state that is no longer invariant to all rotations about its midline axis. Importantly, however, the spontaneous nature of the symmetry-break means that the asymmetric state realized by the rod will be symmetric with respect to a specific rotation and will be one of an infinite number of possible states (i.e., there is an infinite number of directions the rod could have buckled into) that are related to each other via a rotation. It is in this way that spontaneous symmetry-breaking reflects a redistribution of symmetry (see e.g., Castellani, 2003; Stewart and Golubitsky, 1992/2011; Strogatz, 1994, for a more detailed discussion of this and similar physical phenomena).

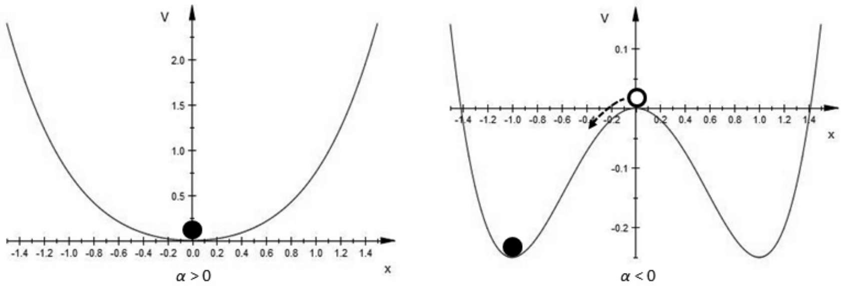


Fig. 5. Eq. (1) plotted as a potential function for (*left*) $a > 0$ and (*right*) $a < 0$. When $a > 0$ the system has one stable solution at $x = 0$, which is invariant with respect to the system's global (higher order) symmetry, namely inversion (i.e., $x \to -x$). When $a < 0$ the system's local symmetry is broken with two stable solutions emerging at $x = \pm\sqrt{a}$ that are not invariant with respect to the global system of Eq. (1) (see text for more details).

Regarding dynamical laws (equations of time-evolving processes), a spontaneous symmetry-break corresponds to solutions of a symmetric law that are not invariant with respect to that symmetry. Take the dynamical (differential) equation

$$\dot{x} = -ax - x^3 \tag{1}$$

This system has global symmetry with respect to inversion (i.e., $x \to -x$), which can be represented by the cyclic group, Z_2. As illustrated in Figure 5, this global (higher order) symmetry is easily discerned when one plots Eq. (1) as a potential function (imagine reflecting the function about the V axis, it would still look the same). When $a > 0$, this global symmetry is locally preserved by the stable fixed-point solution at $x = 0$. Yet, as a is scaled from a positive real number to a negative real number, the local symmetry of the system breaks (a bifurcation occurs), with the global, higher order symmetry of the system being spontaneously redistributed across two new stable states. In more formal terms, the stable solution at $x = 0$ becomes unstable, and two new stable states emerge at $x = \pm\sqrt{a}$, with these two minimal energy states (solutions to Eq. (1)) having less symmetry than the system itself. Of significance, however, the global system remains invariant to inversion even when the system has two differential solutions (broken symmetry) at the local level.

The nonlinear phase transition that characterizes the emergence of Rayleigh-Bénard convection provides another good example of spontaneous symmetry-breaking. In this phenomenon, a relatively thin layer of viscous fluid (e.g., oil) in a horizontal container is heated from below. Applying heat from below creates an energy gradient between the top and bottom of the container and when the energy gradient is low, random collisions between the individual molecules of the fluid are sufficient to dissipate the energy gradient. However, above a critical energy gradient, random collisions between fluid molecules can no longer dissipate the flow of energy and the random motion of the fluid's molecules becomes macroscopically ordered into convection rolls (rotating cells of fluid motion that roll in alternating clockwise and counter-clockwise directions). Figure 6 provides an illustration of the phenomena. Here, the spontaneous symmetry-break can be understood in two ways. First, analogous to Eq. (1), the local symmetry of the system is redistributed from a single homogeneous state to one of two differential states of ordered convection roll motion; a horizontal clockwise/counter-clockwise state or a horizontal counter-clockwise/clockwise state. A second, more conceptual understanding of this spontaneous symmetry-break concerns the organization of the fluid molecules. Prior to the critical point transition, the motion of the fluid's molecules is homogeneously random and unordered, whereby the fluid looks the same from any point of observation. As such, the system is in a high state of symmetry, invariant with regard to spatial and temporal translations. After the critical point transition, however, the ordered, non-homogeneous motion of the fluid's molecules gives rise to a macroscopic behavioral order that has fewer spatial and temporal symmetries; only for very specific spatial and temporal translations does the fluid look the same.

In addition to being a real world analogy of the kind of state bifurcation defined by Eq. (1), the Rayleigh-Bénard convection example highlights how more symmetry is synonymous with less behavioral organization and less

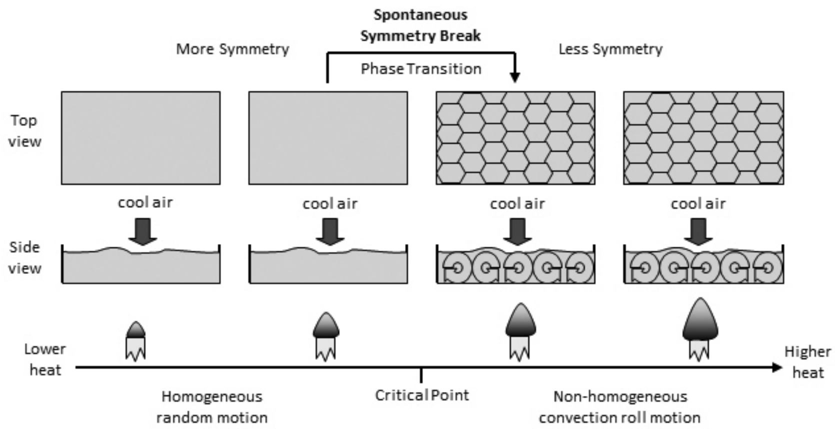


Fig. 6. An illustration of spontaneous symmetry-breaking and Rayleigh-Bénard convection. At a critical energy gradient the molecules of a viscous fluid spontaneously transition from a highly symmetric state of homogeneous random motion to a less symmetric state of non-homogeneous convection roll motion (see text for more details).

symmetry is synonymous with more behavioral organization. Specifically, the state of highly organized convection rolls has less symmetry than the unordered, yet highly symmetric state of random motion that characterizes the system prior to the break in symmetry. The inverse relationship between symmetry and behavioral organization is, of course, intimately related to the idea that symmetry-breaking creates phenomena and, as will be argued here, is key to understanding the behavioral order of social coordination.

As a final example of spontaneous symmetry-breaking, take the phenomena of self-assembled magnetic surface swimmers uncovered by Snezhko, Aranson and Kwok (2006); Snezhko, Belkin, Aranson and Kwok (2009). Images of the basic phenomena are displayed in Figure 7 and depict the self-propelled snake or worm like structures that emerge from a collection of dispersed magnetic, nickel micro-particles in the presence of an alternating magnetic field. The emergence of these self-propelled magnetic snakes is highly robust and can be achieved using a large collection of tiny nickel spheres (magnetic micro-particles) suspended on top of water via surface tension within a glass beaker. When a vertical alternating magnetic field is generated around the beaker using a pair of Helmholtz magnetic coils, the snake like structures emerge due to the collective alignment of the micro-particles to the alternating magnetic field and local surface deformations that result from this alignment response. More specifically, as the nickel micro-particles become aligned with the external magnetic field they produce local deformations on the surface of the water, such that neighboring particles begin to interact and influence each other. These local interactions bring the particles together into segmented chains (with the resulting magnetic moment pointing along the length of the resulting chain), a process that is further promoted by a wave-like motion that emerges along the particle structure (Snezhko *et al.*, 2006, 2009).

Initially, and at low magnetic field frequencies (i.e., below 100 Hz), the snake like structure is more-or-less held in place by four symmetric hydrodynamic vortices that emerge at opposite ends of the snake. In essence, these hydrodynamic vortices operate like miniature engines, pumping liquid along and away from each end of the snake in opposite directions. Beyond a higher, critical frequency, however (i.e., at and above 100 Hz), the symmetry of these hydrodynamic engines becomes unstable and spontaneously breaks (the energy flow at one end of the snake becomes stronger than the energy flow at the other end) and the snake transitions into a self-propelled swimmer. That is, the behavioral organization of the snake transitioning from balanced, non-translational movement in the x-y surface plane, to translational movement in the x-y surface plane. Essentially the magnetic snake goes from being a symmetric, headless-tailless creature, to one that possesses a directional movement oriented front-end and back-end. Accordingly, when one watches the self-propelled swimmers in action the structural asymmetry of the snake's behavior almost appears to be intentional or end-directed (see beim Graben, 2014, for a discussion of the perceived intentionality of magnetic surface swimmers).

Although the behavior of the magnetic snakes is by no means intentional, the self-organized emergence of these self-propelled structures, like Rayleigh-Bénard convection rolls, provides another clear and rather compelling demonstration of how increases in the organization of system behavior is entailed by symmetry-breaking events. Again, breaks in symmetry are synonymous with increases or changes in behavioral order. The beauty of the magnetic snake example is compounded by the fact that the behavioral order that results from the break in the symmetry of the hydrodynamic vortices can also be controlled by adding a glass bead to the system (see Figure 8 and the right images in Figure 7). We discuss this controlled symmetry-break in more detail in the next section. We mention it here, however, as

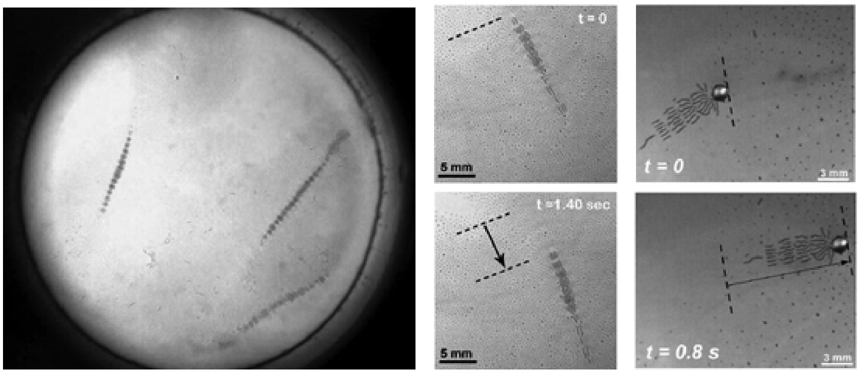


Fig. 7. Self-Assembled Magnetic Surface Swimmers. (*left*) wide and (*middle*) close-up top down views of self-propelled surface swimmers (magnetic snakes) that spontaneously emerge on the surface of a liquid (water) from a collection of magnetic, nickel micro-particles energized by an alternating magnetic field. The self-propelled snakes emerge as a result of spontaneously breaking the symmetry of hydrodynamic surface flows produced at the ends of the snake like structures. (*right*) A bead-snake hybrid swimmer that emerges in a controlled manner due to the bead-head explicitly breaking the symmetry of the surface flows produced by the alternating magnetic field. See text, as well as Figure 8 for more details. The images and description provided here were adapted and paraphrased from Snezhko *et al.* (2009), and with the permission of A. Snezkho from the videos and information available at the time this chapter was written from http://mti.msd.anl.gov/highlights/snakes/experiment.html.

it leads us to consider the question of how systems that undergo spontaneous symmetry-breaking 'choose' which of the many possible asymmetric states available is actually actualized. More specifically, why does the rod buckle this way rather than that way? What determines whether the system defined by Eq. (1) moves from the previously stable state at $x = 0$ to one of the new stable solutions at $x = +\sqrt{a}$ or $x = -\sqrt{a}$? What causes the convection rolls to be ordered in horizontal clockwise/counter-clockwise state or a horizontal counter-clockwise/clockwise state? And, why does a magnetic snake or self-propelled surface swimmer move in one direction versus the other (how does the snake choose which end is the 'front' and which end is the 'back')? The answer to this question is rather simple: some form of asymmetry is required in order for the system to 'choose'. This asymmetry could be extremely small and for a lot of physical systems is determined stochastically via by a small random fluctuation (i.e., noise). For instance, the appearance of the clockwise/counter-clockwise or counter-clockwise/clockwise state during Rayleigh-Bénard convection is typically determined by the amplification of a small random fluctuation in the motion of the oil at the transition point. Similarly, for the magnetic surface swimmers, small differences in particle clustering and composition, or stochastic fluctuations in the strength of the hydrodynamic vortices at each end of the snake quickly amplify as one approaches or moves past the symmetry-breaking, bifurcation point (i.e., at or above the critical 100 Hz frequency of the alternative magnetic current), pushing the snake into one of the two possible directions of propulsion.

As is the case with the magnetic swimmers, however, it is also important to appreciate that the asymmetries that determine one behavioral state or pattern over another need not always be random or stochastic. On the contrary, the direction or state selected during a spontaneous symmetry-break can also be the result of a non-random perturbation or pre-existing asymmetry or bias (Hegstrom and Kondepudi, 1990; Kondepudi and Gao, 1987). Again, such asymmetries are often very small and in some instances may be non-obvious or appear trivial. However, the fact that non-random perturbations or system biases can and often do influence what stable state or behavioral pattern is observed following a spontaneous break in symmetry, means that such behavioral states cannot only be controlled, but can also be intentionally induced.

3.2. *Explicit symmetry-breaking*

Explicit symmetry-breaking refers to a break in symmetry that is due to a process, parameter, or event not entailed by the global or higher order symmetry of the system (Castellani, 2003). Such symmetry-breaking, sometimes called induced symmetry-breaking (Stewart and Golubitsky, 1992/2011; Turvey, 2007), can originate for different reasons and in some instances can break the symmetry of a system in such a way that it makes it impossible to know what underlying symmetry has been broken (the global or higher order symmetry becomes hidden). The outcome of such symmetry-breaking, however, is essentially the same as spontaneous symmetry-breaking in that explicit symmetry-breaking can also create phenomena and bring about greater levels of behavioral order.

As mentioned above with regard to the magnetic swimmers, and illustrated in the right images of Figure 7 and in the sequence of images in Figure 8, breaking the symmetry of the hydrodynamic vortices (surface flows) that emerge at each end of the self-assembled particle chain can be

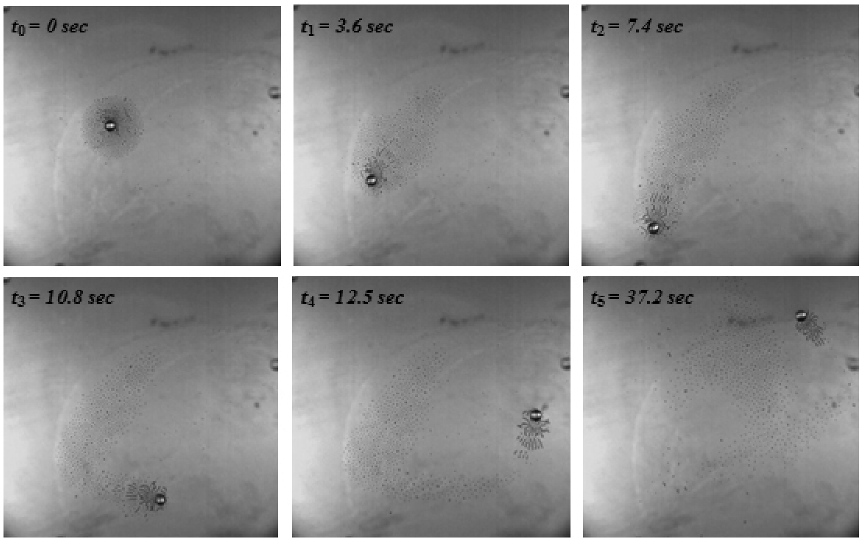


Fig. 8. A series of six static images that illustrate the controlled emergence and swimming behavior of a bead-snake hybrid surface swimmer. As noted in the text and in Figure 7, the bead-snake hybrid swimmer emerges due to the bead-head explicitly breaking the symmetry of the surface flows produced on the surface of liquid water by an alternating magnetic field. The images and description provided here were adapted and paraphrased from Snezhko *et al.* (2009) and with the permission of A. Snezkho from the videos and information available at the time this chapter was written from http://mti.msd.anl.gov/highlights/snakes/experiment.html.

controlled by adding a glass or polystyrene bead to the system. As the snake like particle structure emerges, the bead becomes attached to one end of the snake (forming a bead-snake hybrid) and, in turn, suppresses the vortex flow at that end of the snake. The result is an uncompensated energy flow directly out of what is now the bead-snake's tail end (i.e., the non-bead end), which propels the bead-headed swimmer forward. With respect to the current discussion, the bead explicitly breaks the symmetry of the hydrodynamic vortices and induces a stable direction to the swimmer's translational movement. Interestingly, the bead can be added both before or after a snake emerges and, moreover, can operate to create a self-propelled magnetic swimmer from a stationary snake, even when the alternating magnetic field is below the critical self-propulsion threshold that characterizes the spontaneous symmetry-breaking instability (Snezhko *et al.*, 2009). In short, by explicitly inducing an asymmetry in the system's hydrodynamic surface flows, the bead both creates and instills a more complex and structured level of behavioral organization, one that is even more robust and appears even more alive and intentional than that self-propelled behavior that results from the spontaneous break in symmetry.

As a second example of explicit symmetry-breaking, consider the situation in which some imperfection parameter is imposed on a system. To use an example that we will return to later on in this chapter, lets consider the rhythmic coordination that occurs between the oscillatory leg or arm movements of two visually coupling individuals. The dynamics of such coordination are synonymous with that of a pair of coupled oscillators (Schmidt, Carello and Turvey, 1990; Schmidt and Turvey, 1994), and exhibit a collective state that can be defined by the equation

$$\dot{\phi} = \Delta\omega + \beta \sin(\phi) \tag{2}$$

where ϕ and $\dot{\phi}$ are the relative phase and change in relative phase of the two oscillatory movements, respectively[6]. $\beta \sin(\phi)$ defines the strength of the between-movement coupling, and $\Delta\omega$ is a detuning or imperfection parameter that equals the difference in the natural frequency of the two oscillatory movements (Cohen, Holmes and Rand, 1982; Schmidt, Bienvenu, Fitzpatrick and Amazeen, 1998). As can be seen from inspection of Figure 9, in which Eq. (2) is plotted as a potential function for $-4 < \phi < 4$, when $\Delta\omega = 0$ the system is symmetric with a stable solution at $\phi = 0$, which corresponds to a stable inphase relative phase relationship. Namely, when the two oscillatory movements have the same natural frequency (are symmetric) the solution to the system is spatially symmetric and invariant with respect to inversion (i.e., $\phi \to -\phi$). However, when $\Delta\omega \neq 0$, the symmetry of the system is broken–the function tilts and the stable solution becomes weaker and moves away from $\phi = 0$ as $|\Delta\omega|$ increases, such that $+\phi \neq -\phi$. With regard to a physical system of coupled oscillators or interpersonal rhythmic limb movements, the most significant effect of this explicit symmetry-break is a relative phase lead/lag, where the oscillator or movement with the faster natural frequency leads the oscillator or movement with the slower natural frequency (the synchronous relationship between the two oscillators is also less resistant to perturbations and system noise). Accordingly, when $\Delta\omega \neq 0$, the deviation from perfect inphase

[6]The collective behavior of interpersonal rhythmic coordination is actually best defined by an extended version of Eq. (2), known as the Haken, Kelso, and Bunz (1985) or HKB equation (Haken, Kelso and Bunz, 1985), which can account for antiphase coordination, as well as inphase coordination.

coordination maps onto a differential relation that specifies an identity for each oscillatory movement (i.e., oscillator 1 is different form oscillator 2). There is, of course, still a local symmetry operation that leaves the system invariant, $-(-\phi)$, which is essentially the identity transformation. Furthermore, the higher order symmetry that is broken when $\Delta\omega \neq 0$ can be identified by the fact that the (local) stable solutions for $\Delta\omega < 0$ and $\Delta\omega > 0$, when $|\Delta\omega| < 0 = \Delta\omega > 0$, are qualitatively equivalent and still related by a spatial inversion (i.e., mirror reflection).

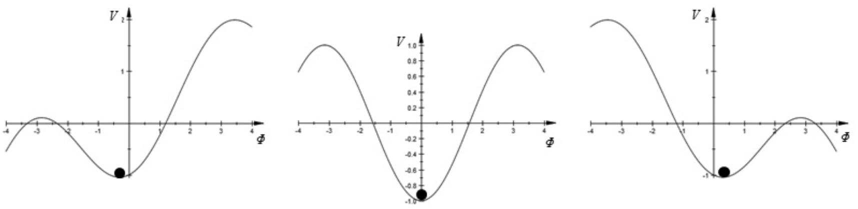


Fig. 9. Eq. (2) plotted as a potential function when $\Delta\omega = -.35$, 0, and .35 from left to right, respectively ($\beta = 1$). When $\Delta\omega = 0$ the system has one stable solution at $\phi = 0$, which is invariant with respect to the system's global (higher order) symmetry, namely inversion (i.e., $\phi \rightarrow -\phi$). When $\Delta\omega \neq 0$, however, the system's local symmetry is broken with the function titling and the stable solution moving away from $\phi = 0$ (see text for more details).

Clearly, the increase in behavioral order that results from the imperfection parameter, $\Delta\omega$, is minimal. However, as we will discuss later, sufficiently large increases in $\Delta\omega$ cannot only result in a differential leader-follower relationship between co-actors performing a rhythmic movement task (Schmidt and Turvey, 1994; Richardson, Marsh, Isenhower, Goodman and Schmidt, 2007b), but can also result in more complex patterns of behavioral coordination to emerge between co-actors, including intermittent coordination (Schmidt and O'Brien, 1997; Richardson, Marsh and Schmidt, 2005) and multirhythmic coordination (Washburn, Coey, Romero and Richardson, 2014). Before detailing this, as well as other examples of how symmetry-breaking influences the structure of social coordination and multiagent activity, let us first review a number of key points that need to be noted in regard to explicit symmetry-breaking.

First, as both of the above examples highlight, an explicit symmetry-break can occur when an introduced or pre-existing asymmetry, imperfection, or differential constraint operates to restructure the symmetry of a system. With this in mind, let us restate Curie's principle again: the (a)symmetry of an effect is defined by the (a)symmetries of the causes that bring about that effect.

Second, nearly all natural or biological systems include asymmetries or imperfections, in that even those systems that one considers to be symmetrical are at best 'nearly symmetrical' or only symmetrical with regard to some idealized or abstract realization of the system. For instance, the human body or face is never perfectly symmetrical, but are close enough that we can consider them as having bilateral or mirror symmetry. Not all of these imperfections or pre-existing asymmetries result in a functionally significant redistribution of symmetry and therefore have little to no effect on the behavioral order expressed. The key is identifying what, when, and how these induced or pre-existing asymmetries or component imperfections operate as explicitly symmetry-breaking factors to restructure or reorder the behavior of a system in a behaviorally relevant and functional manner.

Finally, the parameters that one considers to be explicit symmetry-breaking factors at one spatial or temporal scale, or level of description, may reflect the occurrence of a spontaneous symmetry-break at another scale or level of description. Take the example of the compressed rod. When the rod spontaneously buckles for the first time, randomly actualizing one of the infinitely many possible asymmetric stable states, a bias is introduced into the system, such that future compressions of the rod will likely result in the same (or similar) buckled state. The implication is that explicit symmetry-breaking can reflect a cascade of spontaneous symmetry-breaks. The highly differentiated and imperfect universe we exist in is a perfect example of this. So is the high diversity of human physical, cognitive and interpersonal capabilities, and dispositions. Consistent with our discussion at the end of the previous section on spontaneous symmetry breaking, note that small asymmetries that result from spontaneous symmetry-breaking can also significantly bias a system towards specific asymmetric states. To use a less trivial example than a buckling rod, consider the possibility that the life-dependent preferences in the chirality (i.e., left and right-handedness or mirror symmetry) of molecular and living systems, including human beings, may have been induced by biases established by spontaneous symmetry-breaking events at the atomic or quantum levels (Hegstrom and Kondepudi, 1990).

4. Symmetries of Multiagent Coordination

Hopefully the first half of this chapter has provided a sufficient overview of how the theoretical principles of symmetry and symmetry-breaking provide a powerful and highly generalizable approach for understanding the creation and behavioral (re)order of dynamic phenomena. Armed with this understanding, our focus now turns to describing how these same principles can be employed to investigate, understand, and identify the behavioral order that characterizes multiagent coordination and social action. More specifically, we review a range of different interpersonal and multiagent phenomena that demonstrate how the formal and conceptual language of symmetry and symmetry-breaking provides a general framework for investigating the processes that operate to self-organize the behavioral dynamics of coordinated social activity, and, moreover, how identifying the symmetries and symmetry-breaking factors that define the behavioral order of a particular task or multiagent goal allow us to better understand and uncover the dynamical laws that shape everyday social interaction.

4.1. *Dynamics of solo- vs. joint-action selection*

A good starting place for a discussion of how symmetry and symmetry-breaking operate to define the patterns of coordinated multiagent activity is to consider the behavioral dynamics that characterize the seemingly simple transition between solo and joint action, such as when a pair of co-actors go from moving objects alone to moving or passing objects with or to one another. This action selection process is common to all manner of multiagent task contexts, from completing puzzles and Lego structures together with a friend, to loading or unloading a dishwasher with a family member, to moving office furniture from one location to another with a co-worker. Largely independent of the objects, end-effectors, or agents involved, this action selection process reflects a division between two qualitatively different behavioral modes, whereby socially situated individuals come to organize their behavior with respect to the 'decision' of whether to act alone or together with another individual. Accordingly, solo-action and joint-action reflect generic modes of a multiagent system. More specifically, they represent general order parameters[7] of a multi-agent system.

[7] The term *order parameter* is taken from the field of synergistics (Haken, 1983) and corresponds to the collective state variable(s) with which the behavioral order of a multi-degree-of-freedom dynamical system is defined.

It is important to emphasize that these two collective modes of behavior characterize orthogonal (opposing) directions of system order, both conceptually and physically, in that co-acting individuals cannot perform both modes simultaneously. Indeed, one cannot act together with another individual and at the same time act on their own; you can either pass an object to another actor or move it on your own; similarly, you can either lift an object on your own or with another individual. This point may at first seem self-evident, even trivial. However, it is the orthogonal nature of these modes, whether the specific modes relate to object passing, object moving, or any other bi-ordinal behavior for that matter, that define the symmetry and the symmetry-breaking bifurcations that characterize the dynamic ordering and reordering of solo- vs. joint-action.

To understand how, one need only recognize that orthogonal or opposing modes of behavior tend to interact in a mutually destructive (or constructive) manner, whereby the existence and attractive strength of the two modes covaries. Accordingly, if such modes are represented as different order parameters, for instance as collective variables, ξ_1 and ξ_2, respectively, then the topology of these modes in task space corresponds to two perpendicular axes (i.e., two orthogonal axes crossing to form a 2-dimensional task space). Of particular significance, is that the symmetry of this 2-dimensional task space is isomorphic with the symmetry group of a square, namely the symmetry group D_4. Better yet, it has been shown previously (Haken, 1991, 1983; Kondepudi, 1989) that the dynamics of a system with this kind of D_4 symmetry can be captured using a generic set of differential equations that take the form

$$\begin{aligned}\dot{\xi}_1 &= \lambda_1\xi_1 - B\xi_2^2\xi_1 - C(\xi_1^2 + \xi_2^2)\xi_1 \\ \dot{\xi}_2 &= \lambda_2\xi_2 - B\xi_1^2\xi_2 - C(\xi_2^2 + \xi_1^2)\xi_2\end{aligned} \tag{3}$$

where B and C are positive constants that ensure that the interaction between ξ_1 and ξ_2 is mutually destructive (for most purposes, one can fix B = C = 1), and λ_1 and λ_2 are parameters that determine the growth rates of the amplitudes of ξ_1 and ξ_2, when ξ_1 and ξ_2 are close to zero. One should note that the existence of ξ_1 or ξ_2 as a behavioral mode or state is dependent on λ_1 and λ_2 being greater than zero, respectively. In other words, ξ_1 and ξ_2 only increase from a state equal to zero when their corresponding λ parameter is positive (Frank, Richardson, Lopresti-Goodman and Turvey, 2009; Haken, 1991).

Before detailing how the symmetries of this system entail the behavioral dynamics associated with transitions between solo- and joint-action behavior, let us first highlight the essential properties of this system. To start with, the D_4 symmetry of the dynamical system is visible if one plots its phase portrait when $\lambda_1 = \lambda_2 > 0$ (see Figure 10, top left). In this case, there are four stable fixed points[8],

$$\xi_{1,st} = \pm\sqrt{\frac{\lambda_1}{C}}, \xi_{2,st} = 0 \tag{4}$$

and

$$\xi_{1,st} = 0, \xi_{2,st} = \pm\sqrt{\frac{\lambda_2}{C}} \tag{5}$$

symmetrically defined at positive and negative points along each task axis. This symmetry, however, is broken when $\lambda_1 \neq \lambda_2$. This can be seen from an inspection of the middle and right phase portraits of Figure 10. For moderate differences between $\lambda_1 \neq \lambda_2$, the change in symmetry is quantitative rather than qualitative. Specifically, when λ_1, $\lambda_2 > 0$, but $\lambda_1 < 2\lambda_2$ or $\lambda_2 < 2\lambda_1$ then the fixed points defined by Eq. (4) and Eq. (5) are both stable, but are of different relative strengths (as in Figure 10, top and bottom middle). For example, when λ_1, $\lambda_2 > 0$ and $\lambda_1 < 2\lambda_2$ the stability of the fixed point attractors defined by Eq. (4) is greater than the stability of the fixed points defined by Eq. (5).

Of much more quantitative significance, is the spontaneous symmetry-break that occurs when λ_1, $\lambda_2 > 0$, and $\lambda_1 > 2\lambda_2$ or $\lambda_2 > 2\lambda_1$, whereby the fixed points defined by Eq. (4) or Eq. (5), respectively, become saddle points, leaving stable states only along one system axis (as illustrated in the top and bottom right phase portraits of Figure 10)[9]. In particular, when λ_1, $\lambda_2 > 0$ and $\lambda_2 > 2\lambda_1$, the fixed points along ξ_1, defined by Eq. (4), become unstable saddle points, leaving the fixed points along ξ_2, defined by Eq. (5), as the only stable fixed points. Conversely, when λ_1, $\lambda_2 > 0$ and $\lambda_1 > 2\lambda_2$, the fixed points along ξ_2, defined by Eq. (5), become unstable saddle points, leaving the fixed points along ξ_1, defined by Eq. (4), as the

[8]There is also an unstable fixed point at $\xi_{1,st} = 0, \xi_{1,st} = 0$, which is the only fixed point when $\lambda_1 = \lambda_2 = 0$ (see Figure 10, bottom left). Note that $\lambda_1 = \lambda_2 = 0$ actually reflects the systems highest order of symmetry, in that the solution to the system is invariant with respect to all initial conditions when $\lambda_1 = \lambda_2 = 0$). Thus, the D_4 symmetry of system when $\lambda_1 = \lambda_2 > 0$ already reflects a spontaneous symmetry break in the order of the system and as specified by group theory corresponds to a subgroup of the systems hidden symmetry group, which in this case is the (continuous) orthogonal group O(2) (which is the symmetry group of a circle).

[9]When λ_1 or $\lambda_2 = 0$ the corresponding stable fixed points at Eq. (4) or Eq. (5), respectively, are annihilated at the origin and cease to exist.

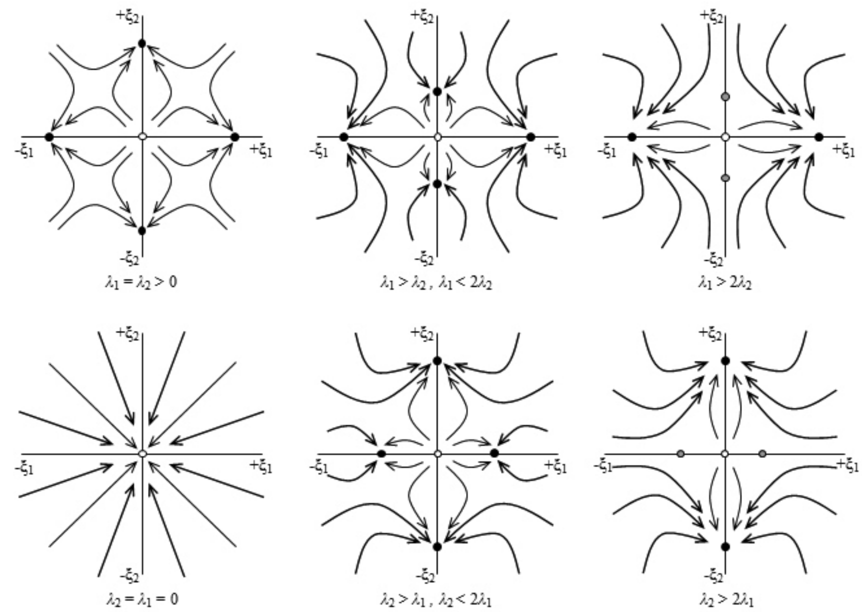


Fig. 10. Phase portraits of Eq. (3) for various settings of λ_1 and λ_2, where symmetric positive and negative solutions (fixed points) along ξ_1 and ξ_2 correspond to solo-action and joint-action, respectively. In each instance $B = C = 1$. Black dots correspond to stable fixed points, grey dots correspond to saddle nodes and white dots correspond to unstable fixed points. Arrowed lines represent exemplar solution trajectories for different initial conditions and indicate the basin of attraction for each stable fixed point (see text for more details).

only stable fixed points. In terms more amenable to the current discussion, if λ_1, $\lambda_2 > 0$, then $\lambda_1 = 2\lambda_2$ and $\lambda_2 = 2\lambda_1$ denote critical points that bring about the spontaneous destruction (or creation) of the stable fixed points along ξ_1 or ξ_2, respectively. In group theoretical terms, the system realizes a D_2 subgroup of the D_4 symmetry group.

From this brief discussion, it should be clear that the key symmetry-breaking parameters in Eq. (3) are λ_1 and λ_2, or, more specifically, the ratio of λ_1 and λ_2. Indeed, λ_1 and λ_2, are the system's control parameters, such that by scaling the ratio of λ_1 and λ_2, when $\lambda_1, \lambda_2 > 0$, one can move the system through the set of spontaneous state bifurcations entailed by the symmetry of the system–each symmetry-breaking bifurcation is defined by a subgroup of the systems higher order symmetry(see footnote 8). Returning to the topic of solo- vs. joint-action selection, what λ_1 and λ_2

might represent will of course be task specific. Generally speaking, though, for physical tasks, such as object moving or passing tasks, the parameters λ_1 and λ_2, or the ratio of λ_1 and λ_2, will correspond to a mode relevant projection of the functional action capabilities of the individuals involved relative to some key physical property. For a two person manual object grasping task, for instance, λ_1 and λ_2 correspond to a collective, mode relevant, projection of each person's hand or arm size relative to object size (Isenhower, Richardson, Carello, Baron and Marsh, 2010; Richardson, Marsh and Baron, 2007a). A crucial point, however, is that if one picks the wrong control parameter(s), then the stable (re)organization of solo vs. joint action will not correspond correctly to the behavioral dynamics defined by the symmetries of orthogonal action modes. Thus, the symmetry and symmetry-breaking transitions entailed by Eq. (3) and the subgroups of D_4 provide a formal method of validating whether one's understanding of the system's parameters and states is correct.

We are now finally in a position to explicate what stable states of behavioral order are possible are for a bi-ordinal solo vs. joint-action action selection task. If we represent the (orthogonal) solo-action and joint-action modes of behavior as the collective variables, ξ_1 and ξ_2, respectively, and for the sake of simplicity, we consider the two symmetric fixed points defined by Eq. (4) to represent equivalent solo-action solutions and, similarly, the two symmetric fixed points defined by Eq. (5) to represent equivalent joint-action solutions, we may then express this as follows:

(1) when $\lambda_1 = \lambda_2$, stable solutions exist along ξ_1 and ξ_2 and are isomorphic with the symmetry group D_4. Solo- and joint-action modes of behavior are therefore equally stable and therefore equally possible. In dynamical systems terms, the system is *multistable*. Either mode may be actualized (but not simultaneously, as noted above) and once the system converges onto a stable solution it will remain there (unless the system is reset to a different initial condition or undergoes a large perturbation).

(2) when $\lambda_1 < 2\lambda_2$ or $\lambda_2 < 2\lambda_1$, stable solutions exist along both ξ_1 and ξ_2 (the system is still multistable), but are of different strength and have differentially sized basins of attraction. Solo- and joint-action modes of behavior are both possible, but with different probabilities (likelihood). Nevertheless, as in (i) above, once the system converges onto either a stable solo- or joint-action solution it will remain there. Thus, if the system finds itself in the weaker of the

two possible behavioral modes due to initial conditions it will still remain there (again, unless the systems is reset to a different initial condition or undergoes a large perturbation). Note that this latter point predicts the occurrence of *hysteresis*, which in short, refers to the presence of history dependent points of state transition. In other words, the control parameter value at which co-actors transition from solo- to joint-action, or from joint- to solo-action will depend, in part, on which mode is currently being performed or has been performed in the past. As such, previous solo action will increase the persistence of future solo action and previous joint-action will increase the persistence of future joint-action.

(3) when $\lambda_1 > 2\lambda_2$ or $\lambda_2 > 2\lambda_1$, stable solutions only exist along either ξ_1 *or* ξ_2, respectively; there are non-stable solutions along the orthogonal axis. That is, solo- or joint-action is possible, but not both (i.e., the probability of one of the two behavioral modes is 1). Accordingly, the system should exhibit a *nonlinear phase transition* between states as the task specific control parameter is scaled beyond or below a value that corresponds to $\lambda_1 > 2\lambda_2$ or $\lambda_2 > 2\lambda_1$. In more general terms, if the system was in a behavioral state that was stable before $\lambda_1 > 2\lambda_2$ or $\lambda_2 > 2\lambda_1$, but is no longer stable after $\lambda_1 > 2\lambda_2$ or $\lambda_2 > 2\lambda_1$, then the system will spontaneously switch to the one stable behavioral mode that still exists. Thus, if the system was in a stable solo-action state before $\lambda_2 > 2\lambda_1$, then it will spontaneously transition to the stable joint-action state after $\lambda_2 > 2\lambda_1$. Conversely, if the system was in a stable joint-action state before $\lambda_1 > 2\lambda_2$, then it will transition to the stable solo-action state $\lambda_1 > 2\lambda_2$.

Research and modeling by Richardson and colleagues (Frank *et al.*, 2009; Isenhower *et al.*, 2010; Richardson *et al.*, 2007a) has validated these symmetry derived patterns of behavioral order with respect to the behavioral dynamics and mode transitions that occur during a two-person object grasping and movement task. There is little need to detail the specific findings of these studies here, as the behavioral ordering of the co-actor's task performance exactly match the symmetry-breaking bifurcations predicted above. Instead, we end this section by reiterating that because the predictions of behavioral order specified in Eq. (3) are derived from symmetry considerations, they are largely independent of the particular solo- vs. joint-action task being considered. That is, these predictions define the behavioral dynamics of any solo- vs. joint action selection task. In fact, the behavioral order and symmetry-breaking bifurcations defined above, apply to any task that entails two mutually destructive or constructive action modes. Thus, they are equally applicable to orthogonally opposed solo-action modes of behavior (e.g., one-hand vs. two-hand grasping or lifting) or orthogonally opposed joint or multi-agent modes of behavior (e.g., solo- vs. joint attacking/defending; two-person vs. three-person lifting and moving). Indeed, such a distinction is only necessary in terms of describing the behavioral task of interest, as the group theoretic explanation is by definition symmetric across a transformation of isomorphic behavioral modes. In other words, Eq. (3) is not task specific, but symmetry specific, and can therefore be generalized to any task that has two mutually destructive collective variables and is isomorphic with the dihedral group D_4. To belabor the point a bit more, the creation and/or destruction of solo- and joint-action is a natural consequence of spontaneous symmetry-breaking, lawfully entailed by the symmetry of orthogonal task modes.

4.2. *Symmetric Hopf bifurcation theory and rhythmic multiagent coordination*

As a second example of how symmetry considerations and the theory of spontaneous symmetry-breaking can provide a deep understanding of the behavioral dynamics that emerge and constrain multiagent coordination, we consider the effects of system symmetry on the behavioral patterns of coordination that are possible between identical and symmetrically coupled rhythmic movements. As a starting point, consider a system of two identical and symmetrically (bi-directionally) coupled oscillators. As mentioned above, this would correspond to a pair of individuals (of similar height and build) coordinating oscillatory (periodic) movements of their respective right or left legs, arms, or body in the visual presence of one another (Schmidt *et al.*, 1990). Similarly, consider a pair of individuals coordinating their rocking movements when seated side-by-side in identical rocking chairs (Richardson *et al.*, 2007b). For such systems, there are only two possible patterns of stable behavioral coordination that naturally exist. The first, and more stable of the two patterns is inphase coordination, which corresponds to the two oscillators moving together in perfect synchrony. The symmetry of this pattern of coordination preserves the symmetry of the system (i.e., two identical and symmetrically coupled oscillators) and, thus, is invariant with respect to inversion. In other words, the observed pattern of coordination would be invariant to a spatial permutation or interchange of oscillator (agent) 1 and oscillator (agent) 2. The other possible pattern is antiphase coordination, which corresponds to the two oscillators being phase locked half a period out of phase with respect to each other (i.e., as one oscillator is beginning a new cycle, the second oscillator is exactly half way through its current cycle). Note, that this second pattern reflects a state of broken symmetry, in that the pattern of coordination is no longer invariant to a purely spatial permutation or interchange of oscillator 1 and 2. That being said, antiphase coordination is still very much entailed by the symmetry of the system and does not correspond to a state of no symmetry, but rather is symmetric (invariant) with respect to the spatiotemporal transformation that permutes the two oscillators/movements and shifts the phase by half a period (Collins and Stewart, 1994; Kelso, 1995)(see the next section and below for more details about the spatial and temporal symmetries of the coupled oscillators).

As an example of the effects of symmetry and symmetry-breaking on the patterns of coordination that can emerge between systems of coupled identical oscillators, the two-component system just described is elegant, but rather simple. Things quickly become more complex, however, as we increase the number of oscillator movements (or agents) involved in a rhythmic or behavioral coordination task. Take for example, the most minimal increase in the number of oscillators that make up a system of symmetrically coupled identical oscillators, that is, from two to three. What patterns of coordination are possible for this system and how are these patterns defined by the symmetry of the system? This question can be answered using the group theoretical approach to dynamical systems with symmetry developed by Golubitsky and Stewart (1985); Stewart (1986). This approach centers on the theory of symmetric Hopf bifurcation, which in simple terms, predicts the periodic or spatiotemporal solutions that can spontaneously arise from a symmetric network of coupled nonlinear oscillators, as well as the transition between different behavioral patterns in terms of symmetry-breaking bifurcations. Unfortunately, a formal discussion of the theory of symmetric Hopf bifurcation is well beyond the scope of this chapter and we refer the reader to Golubitsky and Stewart (1985); Stewart (1986); Golubitsky and Stewart (2003) for a technical introduction to the theorem (for a more accessible review, see Collins and Stewart (1993, 1994) and Stewart and Golubitsky (1992/2011)). However, understanding these details is not necessary given that the predictions are expressed in group-theoretic terms (i.e., with the use of symmetry groups). Furthermore, like symmetry-based approaches in general, the approach is independent of the specific features or intrinsic dynamics of the component oscillators that make up the network. Thus, the approach provides a system independent formalism for predicting the patterns of behavior for any network of identical and symmetrically coupled oscillators, one based entirely on how the symmetries of the patterns observed (the symmetry of effects) relates to the symmetries of the coupled oscillator network itself (the symmetry of the causes) (Golubitsky, Stewart, Buono and Collins, 1999; Collins and Stewart, 1994).

With the latter statements in mind, we return to the question of what patterns of coordination are possible for a system of three symmetrically coupled identical oscillators and attempt to unpack these patterns by means of the symmetry groups that define such a system. To do this, we borrow the descriptive formalism employed by Collins and Stewart (1994), in which the state of an oscillator i at time t can be defined as a (vector) variable $u_i(t)$. Accordingly, the collective state of the entire system, $U(t)$ is a function of each component oscillator $u_i(t)$, such that for a system of three coupled identical oscillators

$$U(t) = (u_1(t), u_2(t), u_3(t)) \tag{6}$$

Given this formalism, it is perhaps immediately obvious that one possible collective state of $U(t)$ would be perfect inphase synchrony, such that

$$u_1(t) = u_2(t) = u_3(t) \tag{7}$$

for all time t. In this case, the behavior of each oscillator would be identical and, thus, $U(t)$ would be spatially symmetric. More specifically, $U(t)$ would be invariant to the permutation (interchange) of each component oscillator $u_i(t)$, whereby

$$\begin{aligned} U(t) = \quad & (u_1(t), u_2(t), u_3(t)) = (u_2(t), u_3(t), u_1(t)) \\ = \quad & (u_3(t), u_1(t), u_2(t)) = (u_1(t), u_3(t), u_2(t)) \\ = \quad & (u_2(t), u_1(t), u_3(t)) = (u_3(t), u_2(t), u_1(t)) \end{aligned} \tag{8}$$

This symmetric relationship is captured by the perturbation group S_3, which as we illustrated back in Figure 3, is isomorphic with the dihedral group D_3 (i.e., the symmetry group of an equilateral triangle). Thus, the inphase pattern of coordination is simply a preservation of the D_3 symmetry of the three oscillator network as a whole–the D_3 symmetry can be discerned from a graphical representation of the network as a symmetric ring (equilateral triangle) of oscillator components (see Figure 11).

Perhaps equally obvious, is that the oscillators cannot all be antiphase from one another. However, it is possible for one oscillator to be out of phase with the other two. For example,

$$u_1(t) = u_2(t) = u_3(t + T/2) \tag{9}$$

in which case all three oscillators would be phase locked, but oscillator 3 would be half a period, T (i.e., $T/2$) out of phase (antiphase) with respect to oscillator 1 and 2. Here, $(t+T/2)$ is a temporal symmetry of the system, such that $u_1(t)$ and $u_2(t)$ are equivalent to $u_3(t)$ when shifted (transformed) in phase by half a period, $T/2$. Together, Eqs. (7) and (9) reveal that $U(t)$ can possess both spatial and temporal symmetries. A quick comparison of Eqs. (7) and (9) also reveals that the symmetry of the pattern defined by Eq. (9) is of a lower order than the symmetry of the inphase pattern defined by Eq. (7). That is, the pattern of coordination defined by Eq. (9) corresponds to a state of broken symmetry with respect to the higher order, D_3 symmetry, of the system's architecture (the symmetry preserved in Eq. (7)). The broken symmetry corresponds to the fact that oscillators 1 and 2 are still spatially symmetric (interchangeable) in Eq. (9), but oscillator 3 is not, whereby

$$U(t) = (u_1(t), u_2(t), u_3(t + T/2)) = (u_2(t), u_1(t), u_3(t + T/2)) \tag{10}$$

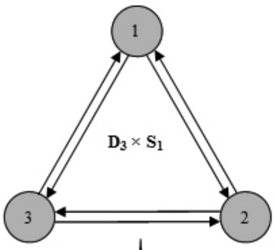
Despite this break in symmetry, the pattern defined in Eq. (9) is still a pattern of coordination that is entailed by the higher order symmetry of the system as a whole. How? Well, the symmetry persevering permutations of Eq. (10) correspond to the cyclic group Z_2, which as we also illustrated back in Figure 3, is a subgroup of D_3. This is significant because it suggests that the symmetry-breaking oscillations that define the possible patterns of a system of three symmetrically coupled identical oscillators could be defined by the subgroups of the system's higher order symmetry group. This is indeed true and, thus, the possible patterns of behavioral order that can be exhibited by the system of symmetrically coupled identical oscillators can be identified entirely via group theoretic calculations (Collins and Stewart, 1994; Golubitsky and Stewart, 2002, 2003).

With the latter knowledge in hand we can now detail the behavioral patterns that are possible for a system of three symmetrically coupled identical oscillators. In short, there are four stable patterns of coordination, plus a state of complete asynchrony (i.e., no stable phase relation) that are predicted from the group-based theory of symmetric Hopf bifurcation developed by Golubitsky and Stewart (1985); Stewart (1986). These patterns are illustrated in Figure 11 and correspond to different isotropy subgroups of $D_3 \times S_1$, where S_1 reflects the temporal symmetry that is entailed by periodic systems, such as the phase shift of $T/2$ defined in Eq. (9). The first is the all-inphase pattern described above, in which all three oscillators are spatially and temporally symmetric. The second pattern is a rotating wave pattern, in which all three oscillators are shifted by $2\pi/3$. For the third pattern, two oscillators have the same waveform and move inphase with one another, with the third oscillator adopting a different waveform and/or phase relationship. Note, that the two-inphase, one-antiphase pattern defined in Eq. (9) is of this type. Finally, for the fourth stable pattern, two oscillators have identical waveforms, but are antiphase with one another (i.e. are shifted by $T/2$ or π), with the third oscillator adopting a different waveform and moving at twice the frequency of the other two oscillators (imagine a person walking with a cane; the persons legs would move in a standard antiphase manner, but the cane would move at twice the frequency of the legs in order to hit the ground with each step; see Strogatz and Stewart, 1993, for a graphical representation of this exact example).

At this point we remind you that because these predictions are derived from symmetry groups, they are independent of the specific details of the nonlinear oscillators employed or the manner of symmetric coupling (i.e., mechanical, neuromuscular, visual, auditory). That is, any system of symmetrically coupled and identical coupled oscillators with a group of symmetries isomorphic with $D_3 \times S_1$ should be constrained to these three generic patterns of behavioral order. This is true for physical, as well as biological systems, including multiagent systems[10]. As evidence of this fact, Ariyabuddhiphongs, Kallen and Richardson (2015) have recently demonstrated how groups of three visually or audibly coupled individuals instructed to drum together (each individual drumming on a drum pad with single drum-stick) are naturally constrained to these generic patterns of coordination. That is, participants can produce all four of the above stable patterns of coordination, including the rotating wave pattern and the pattern in which two individuals drum antiphase, and the third drums at twice the frequency.

A much more compelling example of how the behavioral patterns of a small group of symmetrically coupled agents performing rhythmic or oscillatory movements (or, as in the following case, semi-periodic movements) are

[10]Of course, for biological systems, including multi-agent systems the coupling and intrinsic dynamics of the oscillators will likely be *nearly* symmetric. Thus, small, but accountable deviations may sometimes occur.



Isotropy subgroup	1	2	3	Comments
D_3	A	A	A	All inphase
$\tilde{Z}_3$	A	A + $2\pi/3$	A + $4\pi/3$	Rotating wave
$Z_2(\kappa)$	A	A	B	B could be antiphase with A
$Z_2(\kappa,2T)$	A	A + π	B	B twice the frequency of A

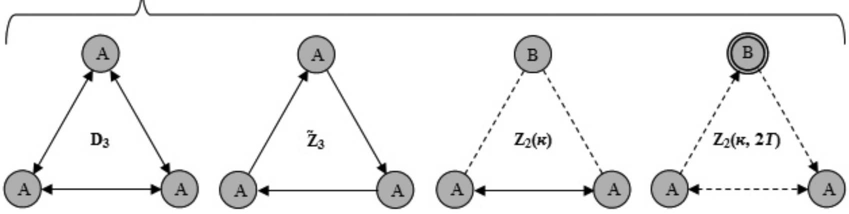


Fig. 11. (*top left*) A graphical representation of a system of three symmetrically coupled identical oscillators. The black arrows represent the symmetric two-way coupling between all oscillators. (*top right*) A table of the generic coordination patterns predicted by symmetric Hopf bifurcation theory (Collins and Stewart, 1994). Letters correspond to the waveform exhibited by an oscillator (same letter, same waveform). κ represents a symmetry transformation of permuting (interchanging) the states of oscillators 1 and 2, while leaving the state of oscillator 3 fixed. $2T$ corresponds to one oscillator having twice the frequency of the other two. (*bottom*). A representation of the coordination patterns detailed in the table. The different forms of arrows (dashed, double, or singled headed) correspond to the different phase relationships that characterize the different coordination patterns. The double lined circle represents the $2T$ oscillator for $Z_2(\kappa, 2T)$.

consistent with those predicted by Golubitsky and Stewart's (1985; 1986) theory of symmetric Hopf bifurcation, comes from a study by Yokoyama and Yamamoto (2011) that investigated the coordination patterns that emerged between individuals playing a 3 vs. 1 ball possession game. The task was a kind of 'monkey-in-the-middle' game, in which three individuals (attackers) positioned in a triangle attempted to pass a ball back and forth between each other in order to prevent a 4th individual positioned in the center of the group from stealing the ball. The rotational movement of each attacker was recorded as the game was played in order to determine the patterns of coordination that occurred between the three attackers. Based on the spatial and temporal symmetries inherent to symmetric Hopf bifurcation theory, the authors predicted that three patterns would likely emerge: (i) a rotating wave pattern, in which the rotational movements of each of the three players exhibited the same waveform and were synchronized with a constant phase difference of $2\pi/3$; (ii) a 2-inphase, 1-antiphase pattern (the authors referred to this as a partial inphase pattern), in which the movements of two players would have the same waveform and be synchronized inphase with one another, while the movements of the 3rd player would exhibit a different waveform and be half a period out of phase (antiphase) with respect to the two inphase players; (iii) a partial antiphase pattern, in which the movements of two players would exhibit the same waveform and be synchronized in an antiphase manner, while the 3rd player would exhibit a different waveform (or no oscillatory movement at all). The results confirmed these expectations, with groups spontaneously adopting all three patterns at various times during the experimental session.

It is important to appreciate that each pattern expected and observed in this 3 vs. 1 ball passing game is isomorphic with one of the generic isotropy subgroups of $\mathrm{D}_3 \times \mathrm{S}_1$, namely $\tilde{\mathrm{Z}}_3$, $\mathrm{Z}_2(\kappa)$, and $\mathrm{Z}_2(\kappa, 2T)$, respectively. Moreover, neither the all-inphase pattern of coordination (i.e., D_3 pattern), nor complete asynchrony were predicted to be observed, as complete symmetry and complete asymmetry represent non-functional organizations with respect to achieving the goal of the multiagent task. In more general terms, the functional behavioral order of the multi-agent system was dependent on symmetry-breaking Hopf bifurcations. We discuss the functional role of symmetry breaking in more detail in the final section of this chapter.

Finally, although we have focused our discussion on the symmetry and symmetry-breaking transitions that define the behavioral order of systems of three coupled identical oscillators, the symmetric Hopf bifurcation theorem developed by Golubitsky and Stewart (1985); Stewart (1986) can be applied to symmetric oscillator networks of other sizes (e.g., $n = 3, 4, 5, 6$ for example). It can even be employed to identify the behavioral possibilities for networks of oscillators that include unidirectional couplings (Collins and Stewart, 1994). In each case, the group theoretic approach is able to identify a generalizable, system independent set of oscillator patterns that could be observed for any system of n coupled identical oscillators. Although it has yet to be investigated, we would therefore predict that the symmetry-breaking transitions that can be derived from the theory of symmetric Hopf bifurcation could therefore account for the functional patterns of coordination that emerge between the rhythmic movements of other small multi-agent groups, for instance groups with 4, 5 or 6 individuals, or for multiagent tasks that require each individual agent to coordinate more than one limb (i.e., both legs or both arms), largely independent of the task being performed (that is, if the system in question is symmetric, or at least near symmetric). Motivated by the successful application of Golubitsky and Stewart's symmetry approach in predicting animal gait patterns (Collins and Stewart, 1993; Golubitsky *et al.*, 1999); (Schöner, Jiang and Kelso, 1990), as well as other multilimb systems (Jeka, Kelso and Kiemel, 1993; Kelso and Jeka, 1992), Harrison and Richardson (2009) have provided initial evidence for this latter claim by demonstrating how the gait patterns observed between individuals walking one behind the other (either visually coupled or mechanically coupled via an interpersonal foam appendage) are not only a direct result of the symmetries that defined a four leg system, but are also consistent with the group-theoretic predictions for quadrapedal systems in general.

4.3. *Detuning and interpersonal rhythmic coordination*

After a careful review of the previous two sections, we hope that you are now convinced of the significance of the group-theoretic approach to dynamical systems with symmetry and the effects of spontaneous symmetry-breaking for understanding the creation, destruction, and (re)organization of coordinated multiagent behavior. Assuming this is so, we now turn to the effects of explicit symmetry-breaking on the behavioral order of multiagent coordination and, in particular, to the explicit symmetry-breaking effects of detuning (i.e., intrinsic frequency differences) on the patterns of interpersonal (two-person) rhythmic coordination.

Recall that two stable patterns of coordination exist for a system of two identical and symmetrically coupled oscillators, namely, inphase coordination, which preserves the symmetry of the system, and antiphase coordination, which corresponds to a state of broken symmetry (Kelso, 1995; Kelso, DelColle and Schöner, 1990). As we have indicated several times already, these exact patterns of coordination are known to define the coordination that occurs between the same (or at least near similar) rhythmic movements of interacting individuals. Indeed, beginning with the foundational work of Schmidt *et al.* (1990), numerous studies have now demonstrated how the coordination that occurs, both intentionally and unintentionally, between identical rhythmic limb and body movements of visual and audibly coupled individuals is constrained without practice (and often without any awareness on the part of the individuals involved) to an inphase or antiphase relative phase relation (for a review see Schmidt and Richardson, 2008).

As noted in the introductory section to explicit symmetry-breaking above, however, differences between the natural frequencies of coupled oscillators (i.e., when $\Delta\omega \neq 0$) introduce an explicit break in the component symmetry of the system (Byblow and Goodman, 1994; Schmidt, Shaw and Turvey, 1993; Sternad, Collins and Turvey, 1995), thus destroying the conical inphase and antiphase patterns in favor of a differential leader/follower relationship. With respect to rhythmic interpersonal coordination, this equates to the individual with the slower intrinsic movement frequency lagging behind the individual with the faster intrinsic movement frequency (Schmidt and Turvey, 1994; Schmidt *et al.*, 1998). Moreover, the magnitude of this effect increases with an increases in the magnitude of detuning, such that as the difference between the natural movement frequencies of coordinating individuals increases, so too does the differentiation of the individuals as phase leader and phase follower.

Clearly, the differential lead/lag relationship that is created by an asymmetry in component movement frequencies is by no means a dramatic example of how explicit symmetry-breaking operates to increase the behavioral order of multiagent coordination. However, once this symmetry-break is introduced, the potential for much more complex and elaborate modes of rhythmic coordination become possible, especially for significantly large magnitudes of detuning (Sternad, Turvey and Saltzman, 1999; Treffner and Turvey, 1993). To understand what patterns are possible and why, one must first understand how the effects of detuning on the patterning of rhythmic coordination are modulated by the strength of the coupling that links the oscillatory movements involved. Note that coupling strength in interpersonal coordination is typically a function of the amount or detectability of visual or auditory information that each individual has about a co-actors movement, the degree to which the individual actors attend to such information, and/or the degree to which the actors intend to coordinate or synchronize their movements (Schmidt and O'Brien, 1997; Richardson *et al.*, 2005, 2007b). In short, increases in coupling strength increase coordination stability and decrease the effects of detuning (i.e., decrease the lead/lag difference). The converse is also true, in that, decreases in coupling strength decrease coordination stability (i.e., increase the variability of coordination) and increase the effects of detuning (i.e., increase the lead/lag different). Key to the patterning and stability of rhythmic interpersonal coordination, therefore, is the relative magnitudes of detuning and coupling strength, such that as the magnitude of detuning increases a stronger coupling strength is required to maintain a stable 1:1 frequency locked inphase or antiphase relationship (Schmidt and Turvey, 1994; Schmidt *et al.*, 1998).

Of particular significance for the current discussion, is that when the coupling strength is sufficiently weak in comparison to the magnitude of detuning, the possibility of stable 1:1 frequency locked synchronization naturally or spontaneously[11] emerging between co-actors becomes highly unlikely and in some instances, impossible (Lopresti-Goodman, Richardson, Silva and Schmidt, 2008; Richardson *et al.*, 2007b). As indicated above, however, this is not to say that such situations result in the complete absence of behavioral coordination, only that they result in the absence of absolute 1:1 frequency locked coordination; the pattern of coordination with the highest degree of symmetry.

So what other patterns of interpersonal behavioral coordination can emerge as a result frequency detuning? The most interesting and somewhat counterintuitive possibility is that frequency detuning can actually induce the appearance of multifrequency or polyrhythmic modes of coordination when the strength of the coupling between co-actors is moderate-to-low (Peper, Beek and van Wieringen, 1995). The simplest example of a multifrequency mode of coordination is 1:2 coordination, where one oscillatory movement completes one cycle for every two cycles of another oscillatory movement. There are, of course, many other *n:m* modes of multifrequency coordination that are possible between two coupled oscillators, such as 1:3, 2:5, 3:8, ..., etc. These modes are specified by the Farey Tree (see Figure 12 left), where movement down the tree is associated with an increase in the complexity or order of the performed frequency ratio[12]. Extended to the circle map that is used to model rhythmic coordination as an emergent frequency relationship between two oscillators based on the ratio of their natural frequencies and coupling strength (Peper *et al.*, 1995; Pikovsky, Rosenblum, Kurths and Hilborn, 2002), one can understand multifrequency coordination as an 'explicitly moderated', spontaneous symmetry-break, such that for moderate to low coupling strengths specific asymmetries in the natural or resonant frequencies of coupled oscillators can induce the emergence of a specific multifrequency coordination mode[13]. As illustrated

[11]Here we use the terms *naturally* and *spontaneously* to refer to situations in which the rhythmic coordination that occurs between co-actors is not intentional, but rather occurs unintentionally and without the actor's overt control.

[12]With respect to group theory, the Farey Tree, like other binary Trees (e.g., the Stern-Brocot tree), as well as fractal sets (i.e., Cantor set) and period doubling maps, can be expressed in such a way that the existence of the different *n:m* ratios are defined by a symmetry group that is isomorphic with the modular group PSL(2,Z) (Vepstas, 2004).

[13]Collins and Stewart (1994) have also demonstrated how certain patterns of multi-

in the right panel of Figure 12, the stability predictions of the circle map illustrated as Arnold tongues allows one to easily identify this prediction, as well as which combinations of frequency detuning and coupling strength allow for the stable emergence of specific multifrequency patterns.

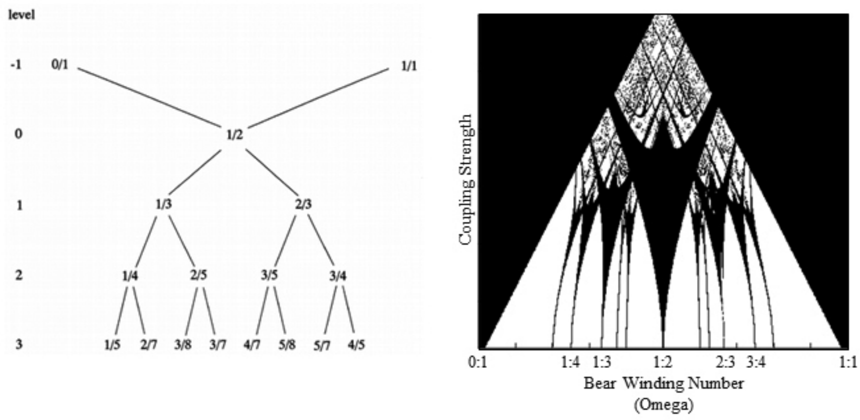


Fig. 12. (*left*) Farey Tree. Generally, higher order ratios are less stable than lower order ratios, and thus are less likely to occur or be sustained over time. (*right*) Arnold tongues for the stable frequency ratios (captured by the bare winding number, omega) predicated by the circle map as a function of coupling strength. The solid black areas are the Arnold tongues and represent the specific intersection of component asymmetry and coupling strength between oscillators that supports the emergence of a number of multifrequency coordination patterns, as well as the differentially sized basins of attraction for each $n{:}m$ mode.

So is this counterintuitive possibility true? Can more complex, multifrequency patterns of rhythmic interpersonal coordination emerge as a result of the explicit and spontaneous symmetry-breaks introduced by changes in frequency detuning and coupling strength, respectively? A recent study by Washburn, Coey, Romero and Richardson (2014) indicates that the answer to this question is yes. In this study, participants were instructed to swing a hand-held pendulum that had a resonant frequency of 1Hz at a comfortable pace while observing an oscillating visual stimulus (the visual stimulus was used as an experimentally controlled proxy for the rhythmic movements of another individual). The frequency of the visual stimulus was manipulated such that its frequency was either equal to, well below, or

frequency coordination, in particular, 1:2 coordination in quadrapedal locomotion (four oscillator systems), are also predicted by Golubitsky and Stewart's group theoretic approach to symmetric Hopf bifurcations.

well above the 1 Hz frequency of the hand-held pendulum (i.e., from .5 and 2 Hz). Somewhat surprisingly, the results demonstrated that the rhythmic movements of the participants did indeed become spontaneously and unintentionally entrained with the oscillatory movements of the visual stimulus at a variety of *n:m* modes, namely at 1:2, 2:3, and even 3:4 modes of multifrequency coordination. Moreover the occurrence of these *n:m* modes was completely consistent with the stability predictions of the circle map, in that they only occurred if (i) the ratio of inherent frequency difference was close to a ratio number that characterizes a specific *n:m* pattern, and (ii) the between movement coupling was weak (i.e., visual and not intentionally defined). Again, the behavioral order observed was a direct consequence of the symmetry-breaking factors that defined the system components and component interactions, in this case, one explicit and one spontaneous.

It is important to appreciate that the participants in the Washburn et al. study were unaware of the multifrequency coordination that occurred between their movements and the movements of the visual stimulus. In other words, the coordination that was observed did not require an 'intention to coordinate' on the part of the participant. In fact, such intentions can make the production of multifrequency coordination very difficult, as an intention to coordinate operates to increases the coupling strength between co-actors (Schmidt and Richardson, 2008). This is why multifrequency patterns of coordination are so difficult to produce intrapersonally (Fontaine, Lee and Swinnen, 1997; Zanone and Kelso, 1992), as the coupling strength that results from the neuro-mechanical links that exist between different limbs of the human body is relatively strong. The multifrequency coordination observed by Washburn *et al.* (2014), was therefore simply an emergent consequence of the physical asymmetries and contextual constraints (i.e., weak coupling strength) that defined the behavioral task. To belabor the point, the symmetry of the effect was written in the symmetry of the causes that brought about that effect. The more general implication, and one that will be exemplified in the other examples to come, is that asymmetries in the intrinsic dynamics of co-acting or socially situated individuals may actually create and promote greater and more complex patterns of behavioral coordination. With respect to interpersonal rhythmic coordination, it is also worth noting that social psychological asymmetries such as group membership or identity (i.e., in-group vs. out-group status) and individual difference asymmetries, such as social competence, can also result in more complex patterns of rhythmic coordination, including lead/lag relationships and intermittent patterns of coordination (Lumsden, Miles, Richardson, Smith and Macrae, 2012; Miles, Lumsden, Richardson and Macrae, 2011; Schmidt, Christianson, Carello and Baron, 1994). Thus, physical component asymmetries are not the only way in which the symmetry of rhythmic interpersonal coordination can be explicitly broken.

4.4. *Functional asymmetries and complementary coordination*

So far we have considered breaks in symmetry that are either defined by the higher order symmetry of a system or system law (i.e., spontaneous symmetry-breaking), or are the result of explicit asymmetries (i.e. imperfections or biases) in the intrinsic component dynamics of a multiagent system. We have said little, however, about the role of symmetry-breaking with respect to the constructive and functional achievement of multiagent task goals, although this was implicit in the examples described above. Thus, in this last section of the chapter we address this question directly and, in particular, detail the constructive, functional and complementary effects of symmetry-breaking with respect to goal directed multi-agent task performance. The reader may note that from this point on, we adopt a more empirical and conceptual approach to the symmetries and symmetry-breaking events that define the behavioral order of multi-agent coordination. However, we fully expect that with a little more work, many of the examples below could by formulated in a group theoretic manner, which may in turn reveal the true, hidden symmetries and laws that underlie these and many other multiagent phenomena.

For cooperative or coordinated multiagent activity, functionally related symmetry-breaks typically correspond to some kind of differential or complementary task role (e.g., leader/follower when weaving through a crowd; driver/navigator during rally racing; or pusher/puller when moving a piece of furniture). These functional or complementary task roles can either exist prior to task performance, say due to some historic or explicitly defined task asymmetry (Byblow and Goodman, 1994) or asymmetric informational constraint, or they can emerge over the course of an interaction, either intentionally or unintentionally. In many instances, these functional or complementary asymmetries denote some form of explicit or induced symmetry-breaking event, or imposed constraint, that is not entailed by some higher order or hidden symmetry. However, it is also possible for functional task asymmetries to occur spontaneously and then be self-sustained via the explicit biases that such spontaneous symmetry-breaking induce (analogues to the buckling rod example described above). Finally, although functional asymmetries are not always necessary for successful task completion, like all symmetry-breaking effects, they nearly always correspond to the generation of new, and often more robust, levels of behavioral organization.

As a preliminary example, let's consider what happens when we explicitly break the social and informational symmetry within a simple rhythmic interpersonal coordination task. This was investigated in a recent study by Vesper and Richardson (2014), in which pairs of participants performed a non-verbal tapping task that required them to synchronize a continuous sequence of taps directed at targets positioned on the surface of a table (i.e., coordinating their tapping movements such that they tapped the same target location at the same time). Four different target locations were employed, with positions equal distance apart directly in front of each participant, and with a single trial sequence involving 256 taps. The key manipulation involved an informationally based asymmetry in task role, whereby one participant was designated as the Leader, and received informational signals about upcoming target locations, while the other participant took the role of Follower, and did not receive this target location signal. An occluding screen was employed so that participants could not see each other's face, eyes, or head, but still had mutual bidirectional visual information allowing them to see each other's complete tapping movement (see Figure 13 left).

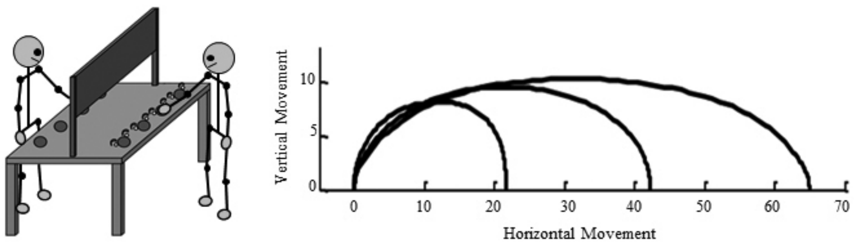


Fig. 13. Experimental setup for the synchronous tapping task investigated by Vesper and Richardson (2014). Average movement trajectories of the Leader as a function of the horizontal target distance to be moved. See text for more details.

Of particular relevance to our discussion here was finding that Leaders involuntarily tapped with a movement amplitude that was scaled to the distance of the target to be tapped, thereby emphasizing for the Follower differences between correct targets and possible alternatives. This effect was not observed in two baseline conditions in which both participants had information about upcoming target locations or the follower could not see the full trajectory movements of the Leader. From the symmetry perspective, the (a)symmetry of the movements produced was a direct result of the (a)symmetry of predefined task roles. That is, the explicit break in the symmetry of the task role resulted in an increase in the functional order of the Leader so as to support the interpersonal task goal (i.e., tap in synchrony). In most instances, the Leader was completely unaware that they were behaving in such a way, indicating that the communicative intent of the Leader's movement was a natural and self-organized consequence of the role induced asymmetry. Lastly, it should be noted that the asymmetry in the Leader's movement trajectories that resulted from the experimentally induced break in task role could only have occurred given the symmetry of the task goal (to move in synchrony)–the dualistic nature of symmetry and asymmetry reveals itself once more.

Vesper and colleagues have uncovered similar findings with respect to two-person synchronized jumping tasks (Vesper, van der Wel, Knoblich and Sebanz, 2013), in which induced asymmetries in the distance to be jumped resulted in a reorganization in the jumping action of the person who had the shortest distance to jump. More specifically, when two co-acting individuals were instructed to jump to different target locations at the same time, individuals with the shortest jumping distance spontaneously modulated their movement dynamics to ensure synchronous coordination.

Together, these tapping and jumping studies provide great examples of how a-priori induced asymmetries in the task role or the availability of task relevant information can operate to functionally structure the behavioral order that occurs between coordinating individuals. Yet, we are still left with the question of whether functionally related complementary roles can emerge as a result of an explicit or spontaneous break in symmetry. With regard to explicit symmetry-breaking in the context of a physical joint action task, one example of how constructive asymmetric movement patterns and complementary participant roles can emerge comes from a study by Bosga, Meulenbroek and Cuijpers (2010) that examined the dynamics of an interpersonal balance board task. The results revealed that pairs learned to stabilize the interpersonal balance board by imposing a differential leader-follower type solution, with the intra-personal control of one individual operating in a subordinate and compensatory role with respect to the control of their co-actor. A more recent example, one that better lends itself to a formal description of how explicit symmetry-breaking parameters can induce more functionally organized behavioral order during multiagent coordination, stems from a study by Richardson and colleagues (Richardson, Harrison, Kallen, Walton, Eiler, Saltzman and Schmidt, 2015; Eiler, Kallen, Harrison and Richardson, 2013) examining the behavioral dynamics of a two-person collision avoidance task. For this task, participant pairs were instructed to perform a repetitive targeting task in which they each moved a computer stimulus back and forth between sets of diagonally opposed target locations without the stimuli colliding into each other. Each participant in a pair stood facing a 50 inch computer monitor and controlled the computer stimulus (a small red dot) using a motion-tracking sensor (see Figure 14 top right). The targets were large squares positioned in each of the four corners of the monitor, with one participant moving their stimulus between the bottom-left and top-right target set, and the other participant moving their stimulus between the bottom-right and top-left target set. Each monitor displayed the real-time motion of the participant's own stimulus, as well as the motion of their co-participant's stimulus. Pairs received 1 point for completing a 40 s trial without colliding and the goal was to achieve a score of 15.

For this task, participant pairs faced a conflict between the natural tendency to synchronize straight-line movement trajectories between the targets in an inphase manner and the fact that such synchronization would result in a collision. The results revealed that pairs overcame this conflict by quickly converging onto a solution that involved complementary task roles, with one participant adopting a more straight-line trajectory between targets and the other participant adopting a more elliptical trajectory between targets (see Figure 14, right). In addition, the participant who adopted the more elliptical trajectory consistently lagged the participant who adopted the more straight-line trajectory by an average of approximately $-30°$. Of particular significance, this asymmetric pattern of behavioral coordination was consistent across pairs and reflected a highly stable pattern of complementary behavioral coordination that enabled participants to concurrently synchronize their movements while avoiding a collision.

Richardson and colleagues hypothesized that these complementary behavioral dynamics resulted from an explicit symmetry-break in the repulsive coupling that prevented collisions. This break not only produced an asymmetry in the ellipticality of the movement trajectories, but also simultane-

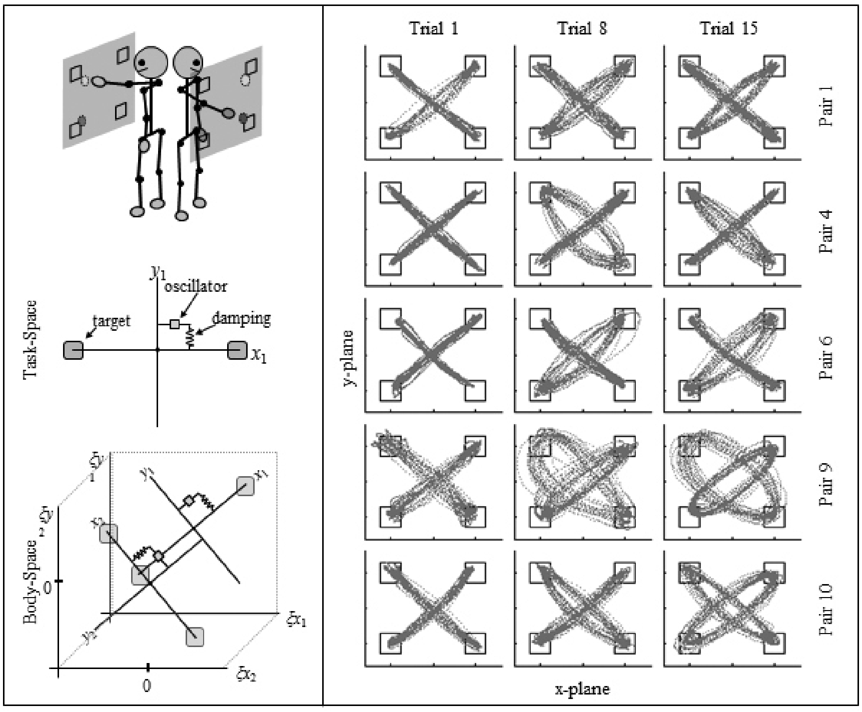


Fig. 14. (*left top*) Representation of the experimental setup and task display employed by Richardson *et al.* (2015). (*right*) Example movement data from trials 1, 8 and 15 for five of the eleven pairs that completed the study. (*left middle*) Abstract representation of the 2-dimensional task space for an individual instructed to perform a simple point-mass (end-effector) rhythmic movement task between two target locations. (*right bottom*) Abstract representation of the joint-action system in which two actors are instructed to coordinate point-masses between targets positioned on orthogonal, 45° angle motion axes with respect to a shoulder-centered coordinate system. For the task space, x_i corresponds to the between target axis of oscillation for a participant, with movement along this axis defined by a limit cycle oscillator. y_i corresponds to orthogonal deviations away from a principal between target movement axis and are therefore defined by a simple damped mass-spring function. ξ_{xi} and ξ_{yi} correspond to the horizontal (frontal) and vertical (sagittal) dimensions of the task movements with respect to shoulder-centered body-space (see text for further details).

ously allowed participants to synchronize their between target movements at a phase lag, and as such, further increase the margin of safety. To test this hypothesis, Richardson *et al.* (2015) formulated a task model of the behavioral dynamics observed utilizing the following set of equations:

$$\begin{aligned}
\ddot{x}_1 - b_1\dot{x}_1 + c_1x_1^2\dot{x}_1 + k_1x_1 &= \alpha_1(\dot{x}_2 - \dot{x}_1) - \gamma_1(x_1 + y_2)e^{-|x_1+y_2|} \\
\ddot{y}_1 - b_2\dot{y}_1 + k_2y_1 &= \gamma_1(y_1 - x_2)e^{-|y_1-x_2|} \\
\ddot{x}_2 - b_1\dot{x}_2 + c_1x_2^2\dot{x}_2 + k_1x_2 &= \alpha_2(\dot{x}_1 - \dot{x}_2) + \gamma_2(x_2 - y_1)e^{-|x_2-y_1|} \\
\ddot{y}_2 - b_2\dot{y}_2 + k_2y_2 &= -\gamma_2(y_2 + x_1)e^{-|y_2+x_1|} \qquad (11)
\end{aligned}$$

Here, each participant's behavior is modeled as an oscillating point-mass within a two-dimensional task space (i.e., a task space plane). An illustration of the two-dimensional task space for a single participant is illustrated in the middle left panel of Figure 14. The projection of this task space within the body space coordinates of the interpersonal behavioral goal is illustrated in the bottom left panel of Figure 14. In this task space, the x-axis corresponds to the axis of instructed oscillation with a van der Pol oscillator employed to generate a self-sustained oscillation of the point-mass along this between target axis. The y-axis corresponds to deviations away from the oscillatory motion axis, hence a simple damped mass-spring equation was used for y to minimize deviations away from the primary motion axis. Accordingly, in Eq. (11) x_1 and y_1, $\dot{x}_1$ and $\dot{y}_1$, $\ddot{x}_1$ and $\ddot{y}_1$ correspond to the position, velocity, and acceleration of participant 1's end-effector within the task space, and x_2 and y_2, $\dot{x}_2$ and $\dot{y}_2$, $\ddot{x}_2$ and $\ddot{y}_2$ correspond to the position, velocity, and acceleration of participant 2's end-effector within the task space. The parameters k_j and b_j are stiffness and damping coefficients, respectively, and the $c_ix_i^2\dot{x}_i$ expressions are the van der Pol (limit cycle oscillator) escapement functions.

In terms of inter-agent coupling, $\alpha_1((\dot{x}_2) - (\dot{x}_1)$ and $\alpha_2((\dot{x}_1) - (\dot{x}_2)$ are dissipative coupling functions that operate to minimize the difference between each participant's primary oscillation axes (i.e., the between target axes x_1 and x_2) with a strength defined by α_i. Finally, the far right expressions in each equation are repeller functions that act to push the two participants' end-effectors away from each other, at a strength determined by an exponential function of distance and γ_i. It is these latter repeller functions and the corresponding strength parameters γ_1 and γ_2 that are most important for our current discussion, in that scaling γ_1 and γ_2 reveals how the complementary roles that contributed to the task success of pairs (i.e., asymmetry in path ellipticality and deviations from 0° relative phase) was the result of a functional inter-participant asymmetry in the strength of these repeller dynamics. This is best demonstrated by detailing the three ways in which scaling γ_1 and γ_2 can influence the movement trajectories produced by Eq. (11):

(1) If $\gamma_1 = \gamma_2 = 0$, then no motion is created along y_1 or y_2 (i.e. $y_1 = y_2 = 0$). Synchronized straight-line movement trajectories are therefore produced along the primary, oscillatory axes of motion, x_1 and x_2, which is equated with unsuccessful task performance as such trajectories would result in a collision (see Figure 15 left).

(2) If $\gamma_1 = \gamma_2 > 0$, then equivalent motion patterns are created along y_1 or y_2 resulting in elliptical trajectories that are symmetric and synchronized with zero phase lag (see Figure 15 middle). Note that this situation actually results in a stable collision avoidance solution, especially for $\gamma_i \gg 0$. The solution is symmetric, though, both interims of the movement trajectories produced and the state topology of Eq. (11)–the solution is invariant to the permutation of (x_1, y_1) and (x_1, y_1)–and does not include a phase lag. In others words, this solution does not entail the functional asymmetry (complementary roles) observed in the experimental data.

(3) If $\gamma_1 \neq \gamma_2$, an asymmetry in the movement trajectory emerges, as well as a phase lag between the more-elliptical and the more-straight-line trajectory (see Figure 15 right). This asymmetry and phase lag is qualitatively similar to that observed in the experimental data reported above. In fact, by modulating the differential magnitudes of γ_1 and γ_2 Eq. (11) can generate a range of movement trajectory patterns that match the range of coordinated movement patterns exhibited by participants and displayed in the bottom right panel of Figure 14 (see Richardson *et al.*, 2015, for more simulation examples).

It is important to note that pairs in the Richardson *et al.* (2015) study were not allowed to converse during the experiment and discovered the coordination pattern defined by Eq. (11) implicitly. In most instances, the participants in the study were not even aware of the dynamic control strategy that they were employing. This implies that control of 'who-did-what-and-when' and the 'intention' of one or both participants in a pair to adopt a specific role emerged as a direct consequence of the (a)symmetries of the task constraints. Further, the explicit symmetry-break in the respective strengths, γ_i, of the repulsive coupling functions defined for participant 1 (x_1, y_1) and participant 2 (x_2, y_2) is analogous to the explicit symmetry-breaking effects of detuning (i.e., $\Delta\omega$) outline above. This sug-

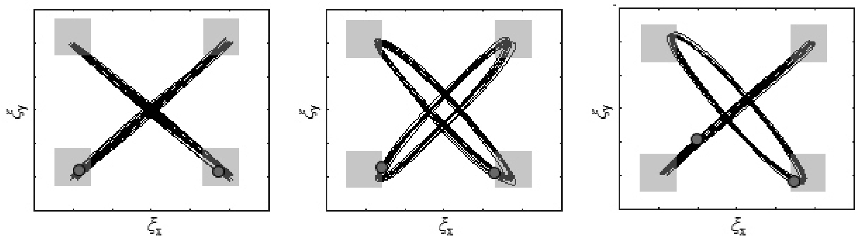


Fig. 15. Examples of how modulating the symmetry and strength of the repulsive coupling parameters γ_1 and γ_2 in Eq. (11) can produce movement trajectories qualitatively similar to those observed between real participants pairs. The simulated time series in each panel were generated using arbitrary initial conditions and the parameter settings $b_1 = 1$, $b_2 = 2$, $k_1 = k_2 = 2\pi$, $c_1 = c_2 = .5$ and $\alpha_1 = \alpha_2 = .5$. (*left*), $\gamma_1 = \gamma_2 = 0$. (*middle*) $\gamma_1 = \gamma_2 = 15$. (*right*) and $\gamma_1 = 20$, $\gamma_2 = 2$. A small amount of Gaussian noise was also added at each time-step (taken from a normal distribution with a mean of 0 and an SD of 5) to all 4 equations in Eq. (11). Solid lines denote movement trajectories; grey dots denote relative movement positions (i.e., relative phase) at an exemplar time step (see text for more details).

gests that functional task asymmetries can induce a break in symmetry in the behavioral dynamics of on-going behavior just as physical or informational differences or imperfections do (Byblow and Goodman, 1994; Sternad, Turvey and Saltzman, 1999). Accordingly, it is likely that the dynamical processes that support complex joint action behavior may well be characterized by the induction and maintenance of asymmetric, inter-agent relations, with explicit changes in the symmetry of joint-action behavior marking the emergence of higher or lower orders of complex and complementary social activity and coordination (Lagarde, 2013; Richardson *et al.*, 2015).

Given these latter points, we feel it is important to provide one more example of how symmetry-breaking bifurcations are synonymous with the emergence of functional, complementary task roles. This last example is also significant because it not only involves more than two agents (is truly a multiagent task), but also concerns a cognitive rather than a physical task goal. It also provides an example of how functional or complementary task roles can emerge from a spontaneous break in symmetry. The example concerns a multiagent binary search task investigated by Roberts and Goldstone (2011). For this task, groups of internet-connected individuals were required to collectively guess a target number between 51 and 100 over a series of guessing rounds. For each round, each individual in the group chose a number between 0 and 50, with the sum of the individual choices equaling the group's guess. After each guess, the groups were informed of whether the collective sum was higher or lower than the target number and this guess-then-feedback process was repeated until the group successfully arrived at the target number. Not surprisingly, groups were able to perform the task successfully and improved over time; groups took fewer rounds to reach the target number as the number of games played increased, with smaller groups taking less time (less rounds) to complete the task than larger groups (i.e., groups of 8 to 10+ individuals). The most interesting result obtained by Roberts and Goldstone, however, was that for large groups individuals in the group tended to adopt differential roles of feedback based reaction, with some individuals adopting a 'reactor' role, whereas other individuals adopted a 'non-reactor' role. In short, reactors were individuals who always changed their individual number choice over repeated attempts/trials, whereas non-reactors were individuals who stuck with the same number choice across repeated attempts/trials. Moreover, the degree and persistence of differential reactivity across participants in the larger groups was positively correlated with a group's overall performance. In other words, an emergent and persistent asymmetry of individual behavior of complementary reactor and non-reactor roles enhanced the collective success of larger groups as a whole. This differential asymmetry in task role was not, however, observed for smaller groups (i.e., groups of 2 to 4 individuals). Rather individuals in these smaller groups all behaved in an equivalent manner across repeated guesses and guessing rounds.

Before elaborating on how the findings of Roberts and Goldstone (2011) provide yet another excellent example of how symmetry-breaking, in this case spontaneous symmetry-breaking, operates to increase the functional organization of multiagent systems, we wish to stress the following: for this multiagent binary search task, cognitive success was not reducible to any one individual group member, nor was it the result of the steady state dynamics of the constituent individuals, but rather was an emergent capacity that arose from the interactions among the individual group members. Indeed, for this task, successful groups converged onto a globally stable pattern of behavior, whereby the stability of this behavioral pattern was dependent on the collective organization of the individuals involved, not the individuals themselves. The same individuals, at a different time and given a different set of initial conditions or responses would have most likely produced a different micro level pattern of individual reactive strategies, and yet exhibited the same overall collective state and group success, both with respect to small and large groups (Theiner, Allen and Goldstone, 2010).

The implication of these latter points is twofold. First, the behavioral state of the multiagent system investigated by Roberts and Goldstone is best defined in terms of a single collective variable or order parameter. Second, the micro level structure of the multiagent system was essentially symmetric with respect to the interchange or permutation of the agents involved. The collective multiagent system would behave the same if one interchanged agent 1 with agent 20, and agent 5 with agent 14 (or any other undergraduate agent from the University of Indiana, where the study was conducted). In fact, it is safe to assume that each individual in a group was more or less identical, in terms of cognitive ability and task understanding.

Armed with the previous points of reference and Curie's principle now firmly instilled (the symmetry of the effect is defined by the symmetry of the causes that underlie that effect) one may readily anticipate why Roberts and Goldstone uncovered what they did. If like us, the weight of this chapter is starting to take on toll on your ability to think clearly, let's break it down.

A system with perfect symmetry must also entail a solution with perfect symmetry; a solution that preserves the symmetry of the system. For the multiagent binary search task, where there was only two qualitative response options (stay with what one did [non-react] or change ones response [react]) this stable solution corresponds to every individual in the group operating in an equivalent manner or, more specifically, as interchangeable and non-persistent reactors and non-reactors. One can think of this solution as similar to the homogenous, less ordered state of the oil in the Rayleigh-Bénard convection system described earlier on in this chapter; the state of the system prior to the symmetry breaking emergence of convection roles. Note that this was the state that characterized smaller groups in the Roberts and Goldstone study.

So what are the other stable states entailed by the symmetry of this multiagent system? Well, there is essentially (qualitatively) only one, a state of broken symmetry, which in this case corresponds to a differential and more ordered grouping of the agents involved. That is, a differential and persistent reactor vs. non-reactor state, which as we noted above is the exact state that larger groups in the Roberts and Goldstone study adopted. Feel free to think of this reactor vs. non-reactor state as a kind of like the more ordered convection role state in the Rayleigh-Bénard convection system, or better yet, as a kind of antiphase state.

You might be saying to yourself at this point in the discussion," ok, so the patterns are predicted by the symmetries of the system, but we are still missing something; what brought about the spontaneous symmetry break as group sized increased?" In other words, what was the control parameter that determined whether groups adopted the symmetric (less ordered) or asymmetric (more ordered) state? Roberts and Goldstone (2011) proposed that the increased difficulty of the task for large groups, compared to small groups, served to selectively pressure an increase in role differentiation and specialization. This task difficulty was a function of feedback related response uncertainty, which in smaller groups (groups with fewer degrees of freedom) was inherently less compared to larger groups. Basically, it was easier for individuals in a 2, 3 or 4 person group to know how (and in which direction) the group would respond as a whole than it was for individuals embedded within a much larger group due to response cancellation. This, of course, makes sense given the symmetry of system. Thus, the spontaneous symmetry break that occurred for the larger groups occurred specifically because it increased the level of behavioral order of the system and, in turn, functionally increased the coordination and performance of the group by reducing uncertainty.

As a side note, this latter example further highlights how system related pressures can spontaneously generate increases in the structure and persistent order of multiagent systems via symmetry breaking, suggesting that symmetry related task difficulty might in fact reflect a general control parameter for systems of collective cognition. In fact, we would conjecture that symmetry defined parameters of task difficulty are likely to be the control parameters that not only defined the types of patterns that are possible for systems of collective cognition, but will also determine when small vs. large group behavior emerges (akin to the above described control parameters that underlie symmetry defined transitions between solo- and joint-action).

5. Conclusion

Our aim with this chapter was ambitious, to convince you that the ordered regularity and behavioral dynamics of coordinated multiagent (social) activity are defined by the symmetries and symmetry-breaking events that characterize an environmentally embedded and contextually situated task goal. Stated more broadly, our aim was to convince you that the formal

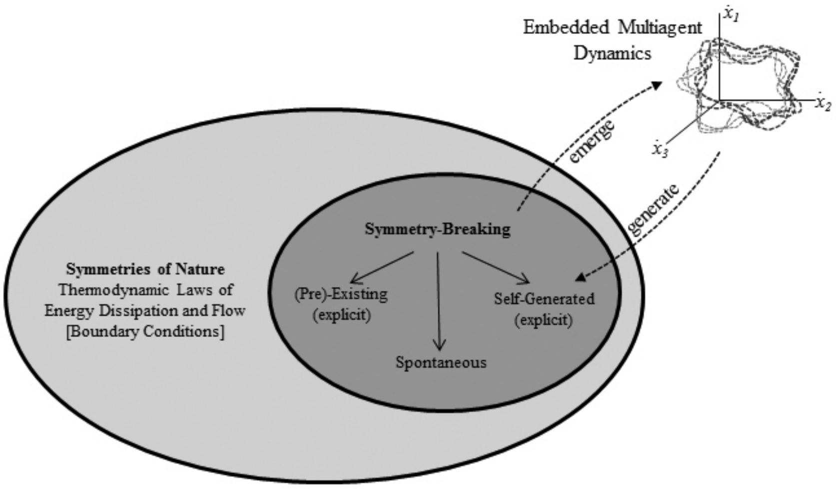


Fig. 16. Reciprocal relationship between symmetry-breaking and the emergence of embedded (contextually dependent) multiagent dynamics. Bound by the behavioral possibilities that the laws of nature prescribe, the behavioral order of multiagent coordination and social activity emerge from symmetry-breaking events that operate to constrain and determined the dynamics of system behavior. In a circularly causal manner, these dynamics further induce (generate) and maintain these brakes in symmetry in order to self-sustain and co-regulate both the short and long term structure of goal directed behavior. In this sense, spontaneous and explicit symmetry-breaking both creates and sustains the existence, complexity and order of multiagent coordination and social activity.

and theoretical principles of symmetry and symmetry-breaking provide a unifying approach to understanding the behavioral order of social coordination and multiagent activity. A synopsis of our argument is illustrated in Figure 16, which is intended to provide an abstract picture of how the symmetry and symmetry-breaking principles discussed here can be used to formalize a lawful and highly generalizable understanding of coordinated social activity.

Finally, we end by noting that although most of the examples discussed here have focused on physical forms of multiagent coordination, the symmetry arguments raised in the current chapter and illustrated in Figure 16 are by no means restricted to these forms of behavior, and as the last example exemplifies, can be applied at any level of behavioral description–from the micro level organization of the human nervous and individual cognition and perceptual-motor activity to the macroscopic organization of groups, societies, and cultures. Indeed, as we have tried to reinforce throughout this chapter, the beauty of understanding behavioral order from a symmetry perspective is that one can develop a common language of explanation that highlights the lawful similitude of behavioral organization across all scales of nature.

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